

Equations – Linear

“ $x^1 = x$ ” is the unknown

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Recall the definition of **Algebra**:

Algebra is arithmetic with letters (aka, applied arithmetic)

and *every* letter represents a number.

Recall the definition of **Algebraic Expression**:

An **algebraic expression** is a numerical expression with letters represent numbers

Now we consider two (2) expressions separated by an equal (“=”) sign:

An **equation** consists of an equal (“=”) sign with an expression on its left-hand side (LHS) and an expression on its right-hand side (RHS):

LHS = RHS

Initially, there is a letter, say “ x ”, on at least one side of the equation which is called the **unknown**. The goal is to find all the values, if any, for “ x ” that make the equation true. This means that when the value for “ x ” is substituted into the equation, the value of the expression of the LHS *equals* the value of the expression on the RHS. This value of “ x ” is called a **solution** of the equation. Consider, for example, for the equation $2x - 4 = 11 - x$. The number $x = 5$ is a solution since

$$2[5] - 4 = 6 = 11 - [5]$$

However, $x = 3$ is NOT a solution since

$$2[3] - 4 = 2 \neq 8 = 11 - [3]$$

The number of solutions an equation possesses depends on its type. We now consider **linear equations** which have “ x ” only raised to the first power: $x = x^1$. Learning how to solve various equations (aka, equalities (“=”)), including linear equations, will require the use of equality properties we now list:

Equality Properties:

Let “a, b & c” be numbers. If $a = b$, then

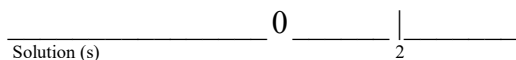
1. $a + c = b + c$
2. $a - c = b - c$
3. $a \cdot c = b \cdot c$
4. $a/c = b/c$; $c \neq 0$

Without letters – except for the unknown

To solve a **linear equation**, we use properties 1 – 4 on both sides until “x” is isolated: $x = \#$. Let’s look at some examples:

1. Solve $x = 2$

This linear equation is already solved; “x” is isolated: $x = 2$. We graph this solution on the horizontal number line:



2. Solve $x - 5 = 3$

Adding “5” to both sides, we obtain

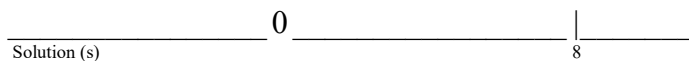
$$x - 5 = 3$$

$$\overbrace{x - 5 + 5}^0 = 3 + 5$$

$$x + 0 = 8$$

$$x = 8$$

Geometrically, we have



3. Solve $4x + 3 = 11$

We have

$$4x + 3 = 11$$

$$4x + \overbrace{3-3}^0 = 11-3$$

$$4x + 0 = 8$$

$$4x = 8$$

$$\frac{4x}{4} = \frac{8}{4}$$

$$x = 2$$

Geometrically, we have

$$\begin{array}{c} \text{Solution (s)} \end{array} \quad \begin{array}{c} 0 \\ \hline 2 \end{array}$$

4. Solve $5 - x = 3x + 20$

$$5 - x = 3x + 20$$

$$5 - \overbrace{x+x}^0 = 3x + 20 + x = 3x + 1x + 20$$

$$5 = 4x + 20$$

$$5 - 20 = 4x + \overbrace{20-20}^0$$

$$-15 = 4x + 0$$

$$-15 = 4x$$

$$-\frac{15}{4} = \frac{4x}{4}$$

$$-\frac{15}{4} = x \left(\text{or } x = -\frac{15}{4} \right)$$

Geometrically, we have

$$\begin{array}{c} \text{Solution (s)} \end{array} \quad \begin{array}{c} \hline -15/4 \end{array} \quad \begin{array}{c} 0 \end{array}$$

5. Solve $3 + (4 - x) = 2(x + 5)$

$$3 + (4 - x) = 2(x + 5)$$

$$3 + 4 - x = 2x + 10$$

$$\overbrace{7 - x + x}^0 = 2x + 10 + x = 2x + 1x + 10$$

$$7 + 0 = 3x + 10$$

$$7 = 3x + 10$$

$$7 - 10 = 3x + \overbrace{10 - 10}^0 = 3x + 0$$

$$-3 = 3x$$

$$-\frac{3}{3} = \frac{3x}{3}$$

$$-1 = x \text{ (or } x = -1)$$

Geometrically, we have

$$\begin{array}{c|c} \text{Solution (s)} & -1 \end{array} \quad 0$$

6. Solve $2(4x + 5) = 2(x + 1) - 4$

$$2(4x + 5) = 2 \overset{\text{Start}}{(x + 1)} \overset{\text{Stop}}{-4}$$

$$8x + 10 = 2x + 2 - 4 = 2x - 2$$

$$8x + \overbrace{10 - 10}^0 = 2x + 2 - 4 = 2x - 2 - 10 = 2x - 12$$

$$8x + 0 = 2x - 12$$

$$8x = 2x - 12 = -12 + \overbrace{2x - 2x}^0 = -12$$

$$6x = 8x - 2x = -12$$

$$\frac{6x}{6} = -\frac{12}{6}$$

$$x = -2$$

Geometrically, we have

$$\begin{array}{c|c} \text{Solution (s)} & -2 \end{array} \quad 0$$

For practice, we'll *check* our “potential solution”:

$$2(\overbrace{4 * (-2)}^{-8} + 5) \stackrel{?}{=} 2(\overbrace{-2 + 1}^{-1}) - 4$$

$$2 * (-3) = -2 - 4$$

$$\stackrel{\text{YES}}{-6 = -6}$$

7. Solve $1 + 4x = 13/4 + x$

$$1 + 4x = \frac{13}{4} + x$$

$$1 + \overbrace{4x - x}^{3x} = \frac{13}{4} + \overbrace{x - x}^0$$

$$3x + \overbrace{1 - 1}^0 = \frac{13}{4} - 1 = \frac{13}{4} - \frac{4}{4} = \frac{9}{4}$$

$$3x = \frac{9}{4}$$

$$\frac{3x}{3} = \frac{\frac{9}{4}}{3} = \frac{\frac{9}{4}}{\frac{3}{1}} = \frac{9}{4} * \frac{1}{3} = \frac{3}{4}$$

$$x = \frac{3}{4}$$

Geometrically, we have

$$\text{Solution (s)} \quad \underline{\hspace{1.5cm}} \quad 0 \quad \left| \quad \underline{\hspace{1.5cm}} \right. \quad \frac{3}{4}$$

For practice, we'll *check* our “potential solution”:

$$1 + 4 * \frac{3}{4} \stackrel{?}{=} \frac{13}{4} + \frac{3}{4} = \frac{16}{4}$$

$$1 + \overbrace{3}^? = 4$$

$$\stackrel{\text{YES}}{4 = 4}$$

8. Solve $0.1x - 1.3 = 2(1.4 - 0.21x)$... *decimal* coefficients

Going to “trade” for *fractional* coefficients

$$\frac{1}{10}x - \frac{13}{10} = 2.8 - 0.42x = \frac{28}{10} - \frac{42}{100}x$$

Going to “trade” for *integer* coefficients

$$100\left(\frac{1}{10}x - \frac{13}{10}\right) = 100\left(\frac{28}{10} - \frac{42}{100}x\right)$$

$$10x - 130 = 280 - 42x$$

$$10x + 42x = 280 + 130$$

$$52x = 410$$

$$x = \frac{410}{52} = \frac{205}{26}$$

Geometrically, we have

$$\text{Solution (s)} \quad \frac{0}{205/26}$$

With letters (more than one) – Literal Equations ... we must be told which letter to solve for

Question: The solution x of the equation $\frac{a}{x} = \frac{b}{a}$ is $x = ?$

Solution:

Step	Equation	Reason
0	$\frac{a}{x} = \frac{b}{a}$	
1	$a^2 = bx$	
2	$\frac{a^2}{b} = x$ OR $x = \frac{a^2}{b}$	

Note: Can not graph the solution set.

Question: The solution x of the equation $a - bx = c(2 - x)$ is $x = ?$

Solution:

Step	Equation	Reason
0	$a - bx = c(2 - x)$	
1	$a - bx = 2c - cx$	
2	$a - 2c = bx - cx$	Group all the x terms together
3	$a - 2c = x(b - c)$	
4	$\frac{a - 2c}{b - c} = x$ OR $x = \frac{a - 2c}{b - c} = \frac{2c - a}{c - b}$	

POWER of Algebra: Solve the following equations:

a. $2x = 3 \Rightarrow x = \frac{3}{2}$

b. $-4x = 7 \Rightarrow x = -\frac{7}{4}$

c. ... There are an infinite number of equations like these. However, consider $ax = b$; $a \neq 0$. We have

$$ax = b \Rightarrow x = \frac{b}{a}$$

Note: We have actually solved an infinite number of equations.
This is the power of algebra!

Additional Linear Equations with “x”:

(1) **Question:** Find the solution of the equation $3 - (4 - 2x) = 3(x + 2) - 4x + 2$

Solution:

Step	Equation	Reason
0	$3 - (4 - 2x) = 3(x + 2) - 4x + 2$	
1	$3 - 4 + 2x = 3x + 6 - 4x + 2$	
2	$-1 + 2x = -x + 8$	
3	$x + 2x = 8 + 1$	
4	$3x = 9$	
5	$x = 3$	

Solution graph: _____|_____3_____

0

(2) **Question:** Solve for x in the equation $2(5x - 3) = 7 - 2x$

Solution:

Step	Equation	Reason
0	$2(5x - 3) = 7 - 2x$	
1	$10x - 6 = 7 - 2x$	
2	$2x + 10x = 6 + 7$	
3	$12x = 13$	
4	$x = \frac{13}{12}$	

Solution graph:

_____|_____13/12_____

0

(3) **Question:** Find the solution x in the equation $\frac{3}{4}x - 2 = \frac{1}{3} + 2x$

Solution:

Step	Equation	Reason
0	$\frac{3}{4}x - 2 = \frac{1}{3} + 2x$	
1	$12\left(\frac{3x}{4} - \frac{2}{1}\right) = 12\left(\frac{1}{3} + \frac{2x}{1}\right)$	Eliminate fractions with common denominator
2	$9x - 24 = 4 + 24x$	
3	$9x - 24x = 24 + 4$	
4	$-15x = 28$	
5	$x = -\frac{28}{15}$	

Solution graph: _____ $-\frac{28}{15}$ _____ | _____
0

(4) **Question:** Solve for x : $\frac{2}{1-x} = \frac{4}{3}$

Solution:

Step	Equation	Reason
0	$\frac{2}{1-x} = \frac{4}{3}$	
1	$2 \cdot 3 = 4(1-x)$	
2	$6 = 4 - 4x$	
3	$4x = 4 - 6$	
4	$4x = -2$	
5	$x = -\frac{2}{4} = -\frac{1}{2}$	

Solution graph: _____ $-\frac{1}{2}$ _____ | _____
0

Note: This equation is a linear “rational” equation since the “ x ” is in the denominator but can be converted to the usual linear format.