

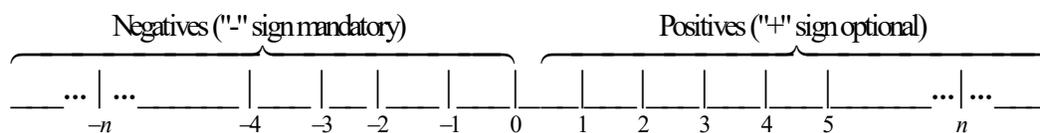
# Real Number System

## Overview

Most of us have used a ruler to measure the length of something:



Each “mark” on a ruler represents a certain number and even when there is no mark shown, a number has been defined. Since numbers are useful in every endeavor, we will define and discuss *all* the numbers needed to fill up an “Infinite Ruler”, called the **Real Number Line**:



The “Integers” are shown above.

We discuss the types of numbers in the following order:

1. **Natural (or Counting) Numbers:**  $N = \{1, 2, 3, 4, 5, \dots, n, \dots\}$
2. **Whole Numbers:**  $W = \{0, 1, 2, 3, 4, 5, \dots, n, \dots\}$
3. **Integers:**  $I = \{\dots, -n, \dots - 5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots, n, \dots\}$
4. **Rational Numbers (Fractions): Rat**

$$Rat = \left\{ \frac{a}{b} \mid \begin{array}{l} \text{Such that} \\ a, b \in I; b \neq 0 \end{array} \right\}$$

**Note:** “ $\in$ ” means “element of”, “member”, ... &  $\{\dots\}$  means a set of elements

**Examples:**  $\frac{1}{2}, -\frac{4}{3}, 0, -2, 4.13, -6.2\overline{54}, \sqrt{9} = 3, \dots$

**Note:**  $N \subset W \subset I \subset Rat$ ; “ $\subset$ ” means “contained in”, “subset of”, ...

**Note:** All fractions have *decimal representations* that

a. Terminate:  $4 = 4.0 = 4.00\dots$

**OR**

b. Repeat:  $3.284284284\dots = 3.284\overline{284}$  **Note:** Three dots imply that the pattern continues

5. **Irrational (Non-fractions): IrRat**

**Note:** All non - fractions have *decimal representations* that DO NOT Terminate or Repeat:

$$\pi = 3.14159265358979\dots$$

$$e = 2.71828182840905\dots$$

$$\sqrt{2} = 1.41421356237310\dots$$

$$2.\overbrace{10}^1\overbrace{100}^2\overbrace{1000}^3\overbrace{10000}^4\dots$$

**Note:** Only the three dots (...) in the last irrational number imply a pattern

The set of *all* these numbers is called the set of **real numbers**.

The real numbers and some relationships between their types are shown below:

## Real Numbers

<p><b>Rat = Rational Numbers</b></p> <p><b>(Fractions)</b></p> <p><math>4, 3.14, -7.44, \frac{11}{7}, -\frac{3}{8}, 13.23\overline{23}, \sqrt[3]{27} = 3, \dots</math></p> <p><i>contains</i></p> <p><b>I = Integers</b></p> <p><math>\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}</math></p> <p><i>contains</i></p> <p><b>W = Whole Numbers</b></p> <p><math>\{0, 1, 2, 3, \dots\}</math></p> <p><i>contains</i></p> <p><b>N = Natural Numbers</b></p> <p><math>\{1, 2, 3, \dots\}</math></p>	<p><b>IrRat = Irrational Numbers</b></p> <p><b>(Non-fractions)</b></p> <p><math>\pi, e, \sqrt{22}, \sqrt[3]{17}, \dots</math></p>
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