College Algebra - Introduction

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FUNdamental Facts for College Algebra

Algebra: Arithmetic with letters.

Note: Arithmetic is *always* trying to lower our grade!

Types (Sets: (Collections, …) of Numbers:

a. **Natural Numbers:** $\{1,2,3,4,5,...\}$

b. **Whole Numbers:** $\{0,1,2,3,4,5,...\}$

c. **Integers**: $\{...-4,-3,-2,-1,0,1,2,3,4,...\}$

d. **Rational (Fractional) Numbers:** $\left\{ \dfrac{a}{b} \middle| a \ \& \ b \ \text{are Integers with} \ b \neq 0 \right\}$

Examples: $\dfrac{3}{5}, -\dfrac{7}{2}, \dfrac{2}{1} = 2,...$

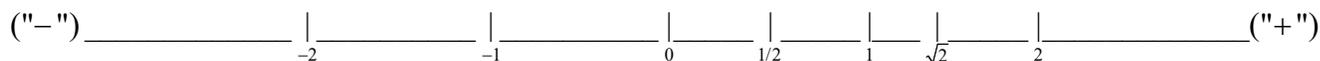Decimal representation repeats: $\dfrac{7}{11} = 0.636363...$

e. **Irrational Numbers:** Non fractional numbers on the Number Line
Decimal representation does NOT repeat: $\sqrt{2} = 1.414213562...$
Examples: $\pi = 3.14159..., e = 2.7182818284590..., \sqrt{7} = 2.645751311,...$

The set of all these numbers, called the **real numbers**, fill up the **Number Line** (think "infinite ruler"):

Number (#) Line: "Real" Numbers – one-part numbers. There are also two-part numbers called **complex numbers** that will be introduced later.

$("-")\underline{\hspace{3cm}} \underset{-2}{|}\underline{\hspace{2cm}} \underset{-1}{|}\underline{\hspace{2cm}} \underset{0}{|}\underline{\hspace{1cm}} \underset{1/2}{|}\underline{\hspace{1cm}} \underset{1}{|}\underline{\hspace{0.5cm}} \underset{\sqrt{2}}{|}\underline{\hspace{1cm}} \underset{2}{|}\underline{\hspace{2cm}}("+")$

Note: NEVER is the Denominator (Bottom) allowed to be zero!

a. $\dfrac{a \neq 0}{0} =$ Undefined

b. $\dfrac{0}{b \neq 0} = 0$

c. $\dfrac{0}{0} =$ Indeterminate ; will be discussed in Calculus

Expressions: Combinations of operations (+,-, …), numbers, letters and grouping symbols ; **NO "="**

Examples:

1. $3x - \dfrac{1}{4}(2 - 5x)$
2. $\dfrac{3x - 4}{1 - x} + 5$
3. $4 - \sqrt{7 - 2x}$
4. $4x - 3y$

Equations: YES "=" (There are many types of equations; we will study some of them)

Equations have a **Left-Hand Side** (LHS) *and* a **Right-Hand Side (RHS)**

LHS Expression = RHS Expression

If the equation has *only* one letter, say "x", the **goal** is to find the values (numbers), when substituted into the LHS *and* RHS, the resulting value are the same, that is, equal. These values, if any, are called **solutions** of the equation.

Example: The number x = 5 is a solution of the equation

$$3x - 2 = 2(4 - x) + 15$$

since

$$3\overset{\frac{5}{?}}{x} - 2 \overset{?}{=} 2(4 - \overset{\frac{5}{?}}{x}) + 15$$

$$15 - 2 \overset{?}{=} 2(4 - 5) + 15$$

$$13 \overset{?}{=} -2 + 15$$

$$13 \overset{YES!}{=} 13$$

The Action Verb for an equation is **SOLVE**!

1. **Equations with one letter (unknown, variable,…)**, say "x":
   Types of equations we will consider:
   a. $3x - 4 = 11$; linear equation ["x" is to the power "1": $x^1 = x$]
   b. $3 - (x + 4) = 2(3 - x)$; linear equation
   c. $x^2 - 2x = 8$; quadratic equation ["x" is to the power "2": x^2]
   d. $|2x - 7| = 3$; absolute value equation: $|?|$
   e. $\sqrt{5x + 6} = 4$; radical equation: $\sqrt{?}$; in general, $\sqrt[n]{?}$
   f. $\dfrac{x}{x - 2} = 2$; rational equation: $\dfrac{?}{?}$
   g. $2^{x-3} = 8$; exponential equation: Base$^{\text{Power}}$
   h. $\log_2(x + 25) = 7$; logarithmic equation: $\log_{\text{Base}}(?)$

2. **Equations with multiple letters**
   a. Examples: $ax + by = c$, $ax = b$, ...
   b. **MUST** be given the "Letter" to "solve for":
      "Letter" $=$ combination of other letters and #'s

Terms, Factors, and Exponents

1. **Terms:** Addition (+) and Subtraction (-)

   Terms: Move across "="

   i. Change sign ("+" to "-" **OR** "-" to "+")
   ii. Keep position

   Example: $2x - 3 = 4 \Rightarrow 2x = 4 + 3$

2. **Factors:** Multiplication (*) and Division (/)

       Factors: Move across "="

         i. Keep sign
        ii. Change position ("Numerator" to "Denominator" **Or** "Denominator" to "Numerator")

       Example: $4x = 5 \Rightarrow x = \dfrac{5}{4}$

Note: Numbers have a lot of properties and it is *best* to memorize them as they are introduced. **Very Important:** If there is a *term* property, there may or may NOT be a corresponding *factor* property!!

3. **Exponents** (Shorthand for multiplication)**:** $\text{BASE}^{\text{Exponent (power,...)}}$
      where Base & Exponent are numbers with some restrictions

        Example: $5^3 = 5*5*5$ (*Simple* Example – much more will be said about exponents)

Order of Operations (Exponents, Factors, & Terms):

1. If grouping symbols ([ ],( ),{ },...) exist, perform the operations from the inner to outer grouping symbols (when fractions are encountered, treat the numerator and denominator separately) as follows
2. Exponents are to be evaluated first
3. Then, the operations of multiplication and division from left to right: Factors $\rightarrow$
4. Finally, the operations of addition and subtraction from left to right: Terms $\rightarrow$

Also,

1. Fractions: work with numerator and denominator separately.
2. Treat square, cube, … roots as if they contain a grouping symbol.