Equations – Introduction

Left-hand Side = Right-hand Side

$$\begin{bmatrix} \text{MATH by Wilson} \\ \text{Your Personal Mathematics Trainer} \\ \text{MathByWilson.com} \end{bmatrix}$$

An **equation** is a statement that the expression on the left-hand side equals ("=") the expression on the right-hand side. The following are a few of the many types of equations:

1. $3x - 4 = 11$; linear equation ["x" is to the power "1": $x^1 = x$]
2. $3 - (x + 4) = 2(3 - x)$; linear equation
3. $x^2 - 2x = 8$; quadratic equation ["x" is to the power "2": x^2]
4. $|2x - 7| = 3$; absolute value equation: $|?|$
5. $\sqrt{5x + 6} = 4$; radical equation: $\sqrt{?}$; in general, $\sqrt[n]{?}$
6. $\dfrac{x}{x - 2} = 2$; rational equation: $\dfrac{?}{?}$
7. $2^{x-3} = 8$; exponential equation: $\text{Base}^{\text{Power}}$
8. $\log_2(x + 25) = 7$; logarithmic equation: $\log_{\text{Base}}(?)$

Notice that each of these equations has the letter "x" in them (actually, other letters may be used); it is called the **unknown**. The *goal* is the **solve** the equation. **Solve** means to find one or more values for "x", if there are any, that make the expression on the left-hand side equal ("=") to the expression on the right-hand side. These values will be called **solutions** of the equation. We are NOT ready to learn how to solve all of the equations above yet, but we can determine if a particular value for "x" is a solution.

1. Show that x = 5 is a solution of the linear equation $3x - 4 = 11$. We just substitute x = 5 into the left-hand side of the equation and see if we get 11.
    Let's see
    $$3x - 4 = 3 * 5 - 4 = 15 - 4 = 11$$
    so x = 5 is a solution.

2. Is x = 7 a solution of $3 - (x + 4) = 2(3 - x)$? Substituting x = 7 in both sides of the equation yields

$$3 - (7 + 4) \overset{?}{=} 2(3 - 7)$$

$$3 - 11 \overset{?}{=} 2(-4)$$

$$-8 \overset{\text{YES}}{=} -8$$

so x = 7 is a solution.

3. Which of the following are solutions of the quadratic equation $x^2 - 2x = 8$:
   a. x = -2
   b. x = 2
   c. x = 4

   We have
   a. x = - 2: YES!
   $$(-2)^2 - 2 * (-2) = 4 + 4 = 8$$
   b. x = 2: NO!
   $$(2)^2 - 2 * (2) = 4 - 4 = 0$$
   c. x = 4: YES!
   $$(4)^2 - 2 * (4) = 16 - 8 = 8$$
   [FYI: quadratic equations have two (2) solutions]

4. Which of the following are solutions of the absolute value equation $|2x - 7| = 3$:
   a. x = -1
   b. x = 2
   c. x = 5

   We have
   a. x = - 1: NO!
   $$|2 * (-1) - 7| = |-2 - 7| = |-9| = 9 \neq 3$$
   b. x = 2: YES!

$$|2*2-7|=|4-7|=|-3|=3$$

    c. x = 5: YES!

$$|2*5-7|=|10-7|=|3|=3$$

5. Which of the following are solutions of the radical equation
$\sqrt{5x+6}=4$:

    a. x = -2

    b. x = 0

    c. x = 2

We have

    a. x = -2: NO!

$$\sqrt{5*(-2)+6}=\sqrt{-10+6}=\sqrt{-10}\neq 4 \ ;$$

        [No negatives under square root until College Algebra –
complex numbers]

    b. x = 0: NO!

$$\sqrt{5*0+6}=\sqrt{6}\approx 2.45 \neq 4 \ ;$$

        [Calculator used to get approximation: $\sqrt{6}\approx 2.45$]

    c. x = 2: YES!

$$\sqrt{5*2+6}=\sqrt{10+6}=\sqrt{16}=4$$

        [We *always* take the positive square root: $\sqrt{16}=4$]

6. Which of the following are solutions of the rational equation
$\dfrac{x}{x-2}=2$:

    a. x = 0

    b. x = 2

    c. x = 4

We have

    a. x = 0: NO!

$$\frac{0}{0-2}=\frac{0}{-2}=0\neq 2 \ ;$$

    b. x = 2: NO!

$$\frac{2}{2-2} = \frac{2}{0} = \text{Undefined} \neq 2 \ ;$$

    c. x = 4: YES!

$$\frac{4}{4-2} = \frac{4}{2} = 2$$

[Note: We can NOT divide out the "4": $\dfrac{4}{4-2}$]

7. Which of the following are solutions of the exponential equation:
    a. x = 0
    b. x = 3
    c. x = 6

We have
    a. x = 0: NO!

$$2^{0-3} = 2^{-3} = \frac{1}{8} \neq 8 \ ;$$

    b. x = 3: NO!

$$2^{3-3} = 2^0 = 1 \neq 8 \ ;$$

    c. x = 6: YES!

$$2^{6-3} = 2^3 = 8$$

8. We will consider logarithmic equations later.

YES, you can probably guess the solutions to some of these equations. But PLEASE concentrate on using the properties of arithmetic/algebra to verify all the calculations as you will NOT be able to guess the solutions of the harder equations we will study later.