Equations – Quadratic: Introduction

MATH by Wilson
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Goal: Solve Quadratic Equations.
Standard Form:

$$ax^2 + bx + c = 0$$

where $a \neq 0, b, c$ are real numbers

Discriminate: $D = b^2 - 4ac$ gives us information about the solutions:

Quadratic Equations have two (2) solutions:

1. **Real Numbers:**
   a. Same number twice: $D = b^2 - 4ac = 0$
   b. Different numbers: $D = b^2 - 4ac > 0$
2. **Complex Numbers** – must occur in conjugate pairs:
   $D = b^2 - 4ac < 0$

Depending upon the values of a,b,c there are different techniques to solve the equation:

1. If $b = 0$: **Square Root Technique**

$$ax^2 + c = 0$$
$$x^2 = -\frac{c}{a}$$
$$x = \pm\sqrt{-\frac{c}{a}}$$

2. If $c = 0$: **Factor Technique**

$$ax^2 + bx = 0$$
$$x(ax + b) = 0$$
$$x = 0 \ \mid \ x = -\frac{b}{a}$$

3. If $a \neq 0$; $b \neq 0$; $c \neq 0$**: Three Techniques**

   a. **Factor** and set each factor equal to zero

   b. **Complete the Square** & then use the Square Root Technique

   c. Use the **Quadratic Formula** (**QF**): $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Help from the past:

1. **Zero Product Fact:**

   $$\square * \triangle = 0 \;\Rightarrow\; \square = 0 \text{ or } \triangle = 0 \text{ or both equal } 0$$

   Example:

   $$x^2 + 2x - 15 = 0 \Rightarrow (x-3)(x+5) = 0$$
   $$\Rightarrow x - 3 = 0 \;;\; x + 5 = 0$$
   $$\Rightarrow x = -5, 3$$

2. **Factor Help:**

   a. $(x+b)(x+d) = x^2 + (b+d)x + bd$

      Example:

      $$x^2 + 2x - 15 = 0 \Rightarrow bd = -15 \;\&\; b+d \text{ MUST be } 2$$

b	d	b+d
15	-1	14
-15	1	-14
3	-5	-2
-3	**5**	**2 (YES!)**

$$\therefore\; x^2 + 2x - 15 = 0 \Rightarrow (x-3)(x+5) = 0$$

b. $(ax+b)(cx+d)=ac\,x^2+(bc+ad)\,x+bd$

Example:

$8x^2+2x-15=0 \Rightarrow ac=8, bd=-5$ & $bc+ad$ MUST be 2

a	c	b	d	bc + ad
2	4	-15	1	-58
2	4	15	-1	58
2	4	-3	5	-2
2	**4**	**3**	**-5**	**2 (YES!)**

$$\therefore 8x^2+2x-15=0 \Rightarrow (2x+3)(4x-5)=0$$

3. **Complete the Square Help:**

Perfect Square:

$$(x+a)^2 = x^2+2a\,x+a^2$$
$$=1*x^2+2a\,x+a^2$$
$$=[x^2\text{- coefficient}]*x^2+[x\text{- coefficient}]*x+[\text{constant}]$$
$$\therefore x^2\text{- coefficient}=1\text{ ; }x\text{- coefficient}=2a\text{ ; constant}=a^2$$

Important:

1. $x^2\text{- coefficient}=1$

2. $x\text{- coefficient}=2a$

$$\frac{1}{2}\left(x\text{- coefficient}\right)=a$$

$$\left[\frac{1}{2}\left(x\text{- coefficient}\right)\right]^2=a^2=\text{constant}$$

Example:

Steps	Example
$x^2 + 2ax = \#$	$x^2 + 6x - 7 = 0 \Rightarrow x^2 + 6x = 7$
$x^2 + 2ax + \left[\frac{1}{2}(x - coeff)\right]^2 = \# + \left[\frac{1}{2}(x - coeff)\right]^2$	$x^2 + 6x + \left(\frac{6}{2}\right)^2 = 7 + \left(\frac{6}{2}\right)^2$
$x^2 + 2ax + a^2 = \# + a^2$	$x^2 + 6x + 9 = 7 + 9$
$(x + a)^2 = \# + a^2$	$(x+3)^2 = 16$
$x + a = \pm\sqrt{\# + a^2}$	$x + 3 = \pm\sqrt{16} = \pm 4$ (2 Solutions!)
$x = -a \pm \sqrt{\# + a^2}$	$x = -3 \pm 4 \Rightarrow x = -7, 1$