Equations – Quadratic: Solve

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Goal: Solve various Quadratic Equations

Example 1: Solve the equation $4\mathbf{x}^2 - 81 = 0$.
Solution:

Note: $\mathbf{a} = 4$; $\mathbf{b} = 0$; $\mathbf{c} = -81$; use the Square Root Technique

Step	Equation	Reason
0	$4\mathbf{x}^2 - 81 = 0$	
1	$4\mathbf{x}^2 = 81$	
2	$\mathbf{x}^2 = \frac{81}{4}$	
3	$\mathbf{x} = \pm\sqrt{\frac{81}{4}} = \pm\frac{9}{2}$	

Graph of solution set:

________ $-9/2$ ________ $|_0$ ________ $9/2$ ____________

Note: There are other ways to solve this and other equations.

Example 2: The solution(s) of the equation $\frac{\mathbf{x}^2}{9}+\frac{1}{16}=0$ satisfies

A) One real solution ("root of multiplicity two")
B) Two real solutions ("distinct solutions")
C) Two complex solutions (NOT conjugate pairs)
D) Two complex solutions (Conjugate pairs)

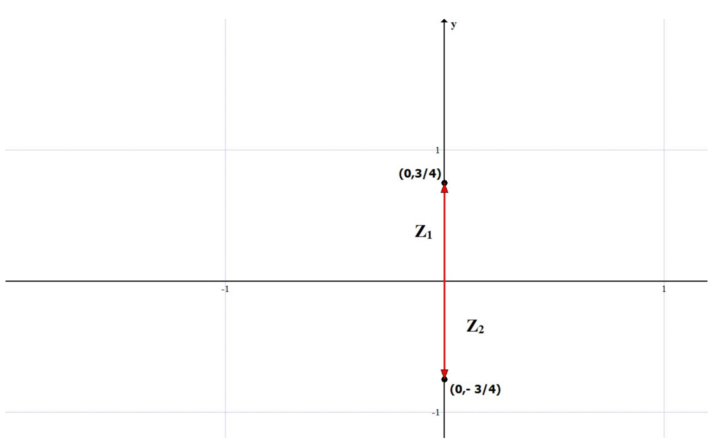
Solution:

Note: $\mathbf{a}=\frac{1}{`9}$; $\mathbf{b}=0$; $\mathbf{c}=\frac{1}{16}$; use the Square Root Technique

Step	Equation	Reason
0	$\frac{\mathbf{x}^2}{9}+\frac{1}{16}=0$	
1	$\frac{\mathbf{x}^2}{9}=-\frac{1}{16}$	
2	$\mathbf{x}^2=-\frac{9}{16}$	
3	$\mathbf{x}=\pm\sqrt{-\frac{9}{16}}$	
4	$\mathbf{x}=\pm\sqrt{\frac{9}{16}}*\sqrt{-1}$	
5	$\mathbf{x}=\pm\frac{3}{4}\mathbf{i}$	

Answer: D

Graph of the solution set:



Example 3: Find the *smallest* solution of the equation $x^2 - 14x + 48 = 0$.
Solution:

Note: $a = 1$; $b = -14$; $c = 48$

Three (3) solutions are given below:

Factor:

Step	Equation	Reason
0	$x^2 - 14x + 48 = 0$	
1	$(x-6)(x-8) = 0$	
2	$x - 6 = 0$ ‖ $x - 8 = 0$ $x = 6$ ‖ $x = 8$	

Complete the Square:

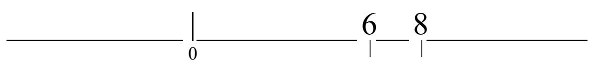
Step	Equation	Reason
0	$x^2 - 14x + 48 = 0$	
1	$x^2 - 14x = -48$	x-coeff: -14 1/2 (x-coeff): -7 $[1/2(\text{x-coeff})]^2$: 49
2	$x^2 - 14x + 49 = -48 + 49$	
3	$(x-7)^2 = 1$	
4	$x - 7 = \pm 1$	
5	$x - 7 = -1$ ‖ $x - 7 = 1$ $x = 6$ ‖ $x = 8$	

Quadratic Formula - QF:

Step	Equation	Reason
0	$x^2 - 14x + 48 = 0$	
1	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
2	$x = \dfrac{-[-14] \pm \sqrt{[-14]^2 - 4[1][48]}}{2[1]}$	

3	$\mathbf{x} = \frac{14 \pm \sqrt{196-192}}{2}$	
4	$\mathbf{x} = \frac{14 \pm \sqrt{4}}{2}$	
5	$\mathbf{x} = \frac{14 \pm 2}{2}$	
6	$\mathbf{x} = \frac{14-2}{2}$ ‖ $\mathbf{x} = \frac{14+2}{2}$ $\mathbf{x} = 6$ ‖ $\mathbf{x} = 8$	

Graph of the solution set:



Example 4: Find the *sum* of the solutions of the equation $10\mathbf{x}^2 - 11\mathbf{x} = 6$.
Solution:

Standard Form: $10\mathbf{x}^2 - 11\mathbf{x} - 6 = 0$ so that $\mathbf{a} = 10$; $\mathbf{b} = -11$; $\mathbf{c} = -6$

Again, three (s) solutions:

Factor:

Step	Equation	Reason
0	$10\mathbf{x}^2 - 11\mathbf{x} - 6 = 0$	
1	$(5\mathbf{x}+2)(2\mathbf{x}-3) = 0$	
2	$5\mathbf{x} + 2 = 0$ ‖ $2\mathbf{x} - 3 = 0$ $5\mathbf{x} = -2$ ‖ $2\mathbf{x} = 3$ $\mathbf{x} = -\frac{2}{5}$ ‖ $\mathbf{x} = \frac{3}{2}$	

Complete the Square:

Step	Equation	Reason
0	$10\mathbf{x}^2 - 11\mathbf{x} - 6 = 0$	
1	$10\left[\mathbf{x}^2 - \frac{11}{10}\mathbf{x}\right] = 6$	x-coeff: $-\frac{11}{10}$ 1/2 (x-coeff): $-\frac{11}{20}$ $[1/2(\text{x-coeff})]^2$: $\frac{121}{400}$
2	$10\left[\mathbf{x}^2 - \frac{11}{10}\mathbf{x} + \frac{121}{400}\right] = 6 + 10 * \frac{121}{400}$	Multiple by $10 * \frac{121}{100}$
3	$10\left[\mathbf{x} - \frac{11}{20}\right]^2 = \frac{361}{40}$	
4	$\left[\mathbf{x} - \frac{11}{20}\right]^2 = \frac{361}{400}$	
5	$\mathbf{x} - \frac{11}{20} = \pm\sqrt{\frac{361}{400}} = \pm\frac{19}{20}$	
6	$\mathbf{x} - \frac{11}{20} = -\frac{19}{20}$ ‖ $\mathbf{x} - \frac{11}{20} = \frac{19}{20}$ $\mathbf{x} = \frac{11}{20} - \frac{19}{20}$ ‖ $\mathbf{x} = \frac{11}{20} + \frac{19}{20}$ $\mathbf{x} = -\frac{8}{20} = -\frac{2}{5}$ ‖ $\mathbf{x} = \frac{30}{20} = \frac{3}{2}$	

Quadratic Formula - QF:

Step	Equation	Reason
0	$10\mathbf{x}^2 - 11\mathbf{x} - 6 = 0$	
1	$\mathbf{x} = \frac{-\mathbf{b} \pm \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}$	
2	$\mathbf{x} = \frac{-[-11] \pm \sqrt{[-11]^2 - 4[10][-6]}}{2[10]}$	
3	$\mathbf{x} = \frac{11 \pm \sqrt{121 + 240}}{20}$	
4	$\mathbf{x} = \frac{11 \pm \sqrt{361}}{20}$	

5	$\mathbf{x} = \frac{11 \pm 19}{20}$	
6	$\mathbf{x} = \frac{11-19}{20}$ $\mathbf{x} = -\frac{5}{2}$ ‖ $\mathbf{x} = \frac{11+19}{20}$ $\mathbf{x} = \frac{3}{2}$	

Graph of the solution set:

________ $-2/5$ ___ | (0) ____________________ $3/2$ ______________

Example 5: Find the *largest* solution $\mathbf{x}_{\text{Large}}$ of the equation $3\mathbf{x}^2 + 2\mathbf{x} - 4 = 0$.

Solution:

Note: $\mathbf{a} = 3$; $\mathbf{b} = 2$; $\mathbf{c} = -4$

Only two (2) solutions below:

Factor:

Doesn't factor "nicely"! **UGLY!**

Complete the Square:

Step	Equation	Reason
0	$3\mathbf{x}^2 + 2\mathbf{x} - 4 = 0$	
1	$3\left[\mathbf{x}^2 + \frac{2}{3}\mathbf{x}\right] = 4$	x-coeff: $\frac{2}{3}$ 1/2 (x-coeff): $\frac{1}{3}$ [1/2(x-coeff)]^2: $\frac{1}{9}$
2	$3\left[\mathbf{x}^2 + \frac{2}{3}\mathbf{x} + \frac{1}{9}\right] = 4 + 3 * \frac{1}{9}$	

3	$3\left[x+\frac{1}{3}\right]^2=\frac{13}{3}$	
4	$\left[x+\frac{1}{3}\right]^2=\frac{13}{9}$	
5	$x+\frac{1}{3}=\pm\sqrt{\frac{13}{9}}=\pm\frac{\sqrt{13}}{3}$	
6	$x+\frac{1}{3}=-\frac{\sqrt{13}}{3}$ $x=-\frac{1+\sqrt{13}}{3}$ $x\approx-1.5352$ ‖ $x+\frac{1}{3}=\frac{\sqrt{13}}{3}$ $x=\frac{-1+\sqrt{13}}{3}$ $x\approx 0.8685$	

Quadratic Formula - QF:

Step	Equation	Reason
0	$3x^2+2x-4=0$	
1	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	
2	$x=\frac{-[2]\pm\sqrt{[-2]^2-4[3][-4]}}{2[3]}$	
3	$x=\frac{-2\pm\sqrt{4+48}}{6}$	
4	$x=\frac{-2\pm\sqrt{52}}{6}=\frac{-2\pm 2\sqrt{13}}{6}$	
5	$x=\frac{-1\pm\sqrt{13}}{3}$	
6	$x=\frac{-1-\sqrt{13}}{3}$ $x\approx-1.5352$ ‖ $x=\frac{-1+\sqrt{13}}{3}$ $x\approx 0.8685$	

Graph of the solution set:

________ −1.54 __________________ | 0 ________ 0.87 ____________

The largest solution is 0.87.

Example 6: Is there a *largest* solution $\mathbf{x}_{\text{Large}}$ of the equation $\mathbf{x}^2+3\mathbf{x}+5=0$?
Solution:

Note: $\mathbf{a}=1$; $\mathbf{b}=3$; $\mathbf{c}=5$

Ugly again; only two solutions:

Factor:

Doesn't factor "nicely"! **UGLY!**

Complete the Square:

Step	Equation	Reason
0	$\mathbf{x}^2+3\mathbf{x}+5=0$	
1	$\mathbf{x}^2+3\mathbf{x}=-5$	x-coeff: 3 1/2 (x-coeff): $\frac{3}{2}$ [1/2(x-coeff)]^2: $\frac{9}{4}$
2	$\mathbf{x}^2+3\mathbf{x}+\frac{9}{4}=-5+\frac{9}{4}$	
3	$\left(\mathbf{x}+\frac{3}{2}\right)^2=-\frac{11}{4}$	
4	$\mathbf{x}+\frac{3}{2}=\pm\sqrt{-\frac{11}{4}}$	
5	$\mathbf{x}+\frac{3}{2}=\pm\frac{\sqrt{-11}}{2}=\pm\frac{\sqrt{11}\,\mathbf{i}}{2}$	
6	$\mathbf{x}+\frac{3}{2}=-\frac{\sqrt{11}\,\mathbf{i}}{2}$ ‖ $\mathbf{x}+\frac{3}{2}=\frac{\sqrt{11}\,\mathbf{i}}{2}$ $\mathbf{x}=-\frac{3+\sqrt{11}\,\mathbf{i}}{2}$ ‖ $\mathbf{x}=\frac{-3+\sqrt{11}\,\mathbf{i}}{2}$	

Quadratic Formula - QF:

Step	Equation	Reason
0	$\mathbf{x}^2+3\mathbf{x}+5=0$	
1	$\mathbf{x}=\frac{-\mathbf{b}\pm\sqrt{\mathbf{b}^2-4\mathbf{ac}}}{2\mathbf{a}}$	
2	$\mathbf{x}=\frac{-[3]\pm\sqrt{[3]^2-4[1][5]}}{2[1]}$	
3	$\mathbf{x}=\frac{-3\pm\sqrt{9-20}}{2}$	
4	$\mathbf{x}=\frac{-3\pm\sqrt{-11}}{2}$	
5	$\mathbf{x}=\frac{-3\pm\sqrt{11}\,\mathbf{i}}{2}$	
6	$\mathbf{x}=\frac{-3-\sqrt{11}\,\mathbf{i}}{2} \quad \Vert \quad \mathbf{x}=\frac{-3+\sqrt{11}\,\mathbf{i}}{2}$	

Graph of the solution set:

There is NOT a largest solution since the solutions are complex numbers.