Quadratic Equations – Applications

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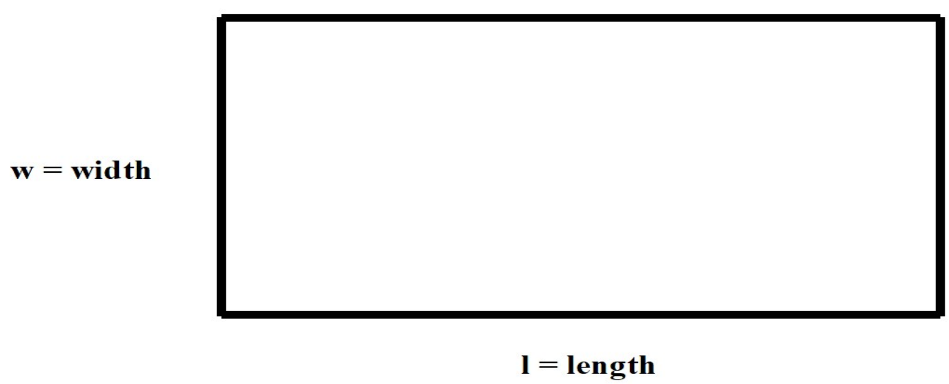
Various Word Problems, Applications, …:

Geometry - Rectangle

(1) Given that the perimeter “P” of a rectangle is 72” and its area “A” is 224 sq “, find its length l and its width w.

Fundamental facts: $\mathbf{P} = 2\mathbf{l} + 2\mathbf{w}$; $\mathbf{A} = \mathbf{l} * \mathbf{w}$



Substitute for the “knowns”:

$224 = \mathbf{l} * \mathbf{w}$
$72 = 2\mathbf{l} + 2\mathbf{w}$

$36 = \mathbf{l} + \mathbf{w}$

$\mathbf{l} = 36 - \mathbf{w}$

Step	Equation	Reason
0	$224 = \mathbf{l} * \mathbf{w}$	1 Equation with 2 Unknowns
1	$224 = (36 - \mathbf{w}) * \mathbf{w}$	1 Quadratic Equation with 2 Unknowns
2	$224 = 36\mathbf{w} - \mathbf{w}^2$	
3	$\mathbf{w}^2 - 36\mathbf{w} + 224 = 0$	

4	$(\mathbf{w}-8)(\mathbf{w}-28)=0$	
5	$\mathbf{w}-8=0$ ‖ $\mathbf{w}-28=0$ $\mathbf{w}=8$ ‖ $\mathbf{w}=28$	
6	$\mathbf{w}=8"$ (smaller of two values) $\mathbf{l}=36-\mathbf{w}=28"$	

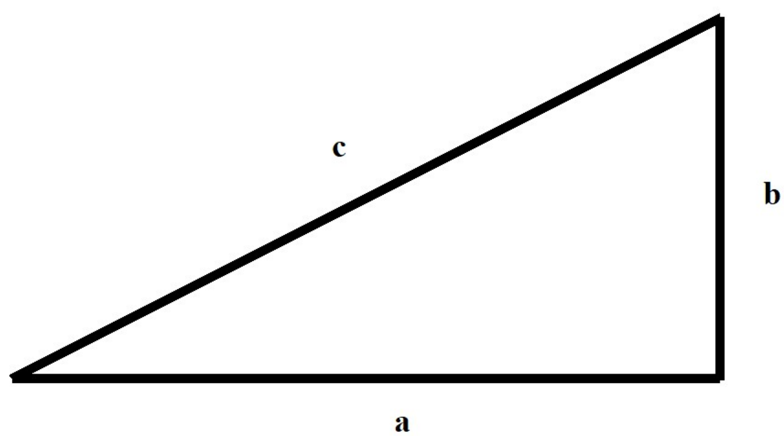
Geometry – Right Triangle (I "smell" the **Pythagorean Theorem**)

In a **right** triangle:

$$(\text{Side}_1)^2+(\text{Side}_2)^2=(\text{Hypotenuse})^2$$

$$\mathbf{a}^2+\mathbf{b}^2=\mathbf{c}^2$$

Note: Hypotenuse is "longest" side



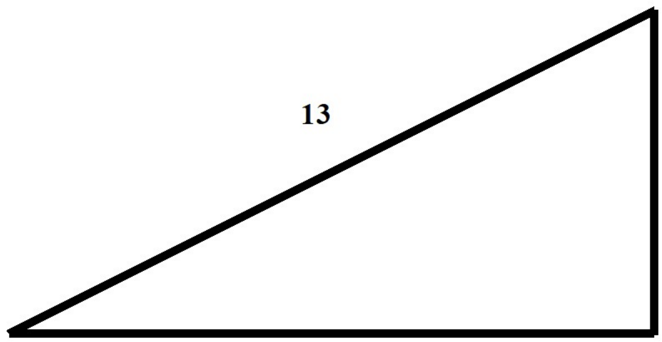
(2) The hypotenuse of a *right* triangle is 13”. If one side is 7” longer that the other side, find the perimeter “**P**” and area “**A**” of this triangle.

Fundamental facts: $\mathbf{P} = \mathbf{a} + \mathbf{b} + \mathbf{c}$; $\mathbf{A} = \frac{1}{2} * \mathbf{a} * \mathbf{b}$

Let $\mathbf{a} = \mathbf{x}$ (one unknown side)

Then $\mathbf{b} = \mathbf{x} + 7$ (other unknown side)

$\mathbf{c} = 13$



Step	Equation	Reason
0	$\mathbf{x}^2 + (\mathbf{x}+7)^2 = 13^2$	Pythagorean Theorem
1	$\mathbf{x}^2 + \mathbf{x}^2 + 14\mathbf{x} + 49 = 169$	
2	$2\mathbf{x}^2 + 14\mathbf{x} - 120 = 0$	
3	$\mathbf{x}^2 + 7\mathbf{x} - 60 = 0$	
4	$(\mathbf{x}-5)(\mathbf{x}+12) = 0$	Factor
5	$\mathbf{x} - 5 = 0$ ‖ $\mathbf{x} + 12 = 0$ $\mathbf{x} = 5$ ‖ $\mathbf{x} = -12$	
6	$\mathbf{a} = \mathbf{x} = 5"$ (positive value) $\mathbf{b} = \mathbf{x} + 7 = 12"$	
7	$\mathbf{P} = \mathbf{a} + \mathbf{b} + \mathbf{c} = 5 + 12 + 13 = 30"$	
8	$\mathbf{A} = \frac{1}{2} * \mathbf{a} * \mathbf{b} = \frac{1}{2} * 5 * 12 = 30$ sq"	

Motion

(3) At the same time when Dr W cycles **north** at a constant rate of 5 mph, Mrs. Dr W cycles **east** at 12 mph. When will they be 52 miles apart?

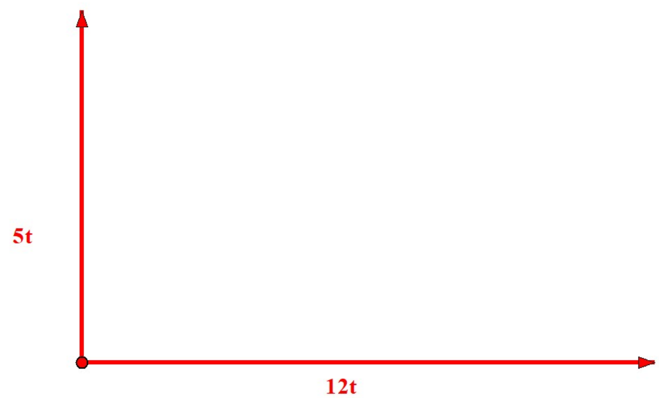
Fundamental facts: Distance = Rate (Constant) * Time

$$\mathbf{d} = \mathbf{r} * \mathbf{t} \; ; \; \mathbf{a}^2 + \mathbf{b}^2 = \mathbf{c}^2$$

Dr W: $\mathbf{d}_1 = \mathbf{r}_1 * \mathbf{t}_1 = 5\mathbf{t}_1$

Mrs. Dr W: $\mathbf{d}_2 = \mathbf{r}_2 * \mathbf{t}_2 = 12\mathbf{t}_1$

Note: $\mathbf{t}_1 = \mathbf{t}_2 \, (\text{call it } \mathbf{t})$; they traveled the same length of time.



Let $\mathbf{a} = \mathbf{d}_1 = 5\mathbf{t}$

Then $\mathbf{b} = \mathbf{d}_2 = 12\mathbf{t}$

$\mathbf{c} = 52$

Step	Equation	Reason
0	$(\mathbf{d}_1)^2 + (\mathbf{d}_2)^2 = 52^2$	
1	$(5\mathbf{t})^2 + (12\mathbf{t})^2 = 52^2$	
2	$25\mathbf{t}^2 + 144\mathbf{t}^2 = 2704$	
3	$169\mathbf{t}^2 = 2704$	
4	$\mathbf{t}^2 = 16$	
5	$\mathbf{t} = \pm 4$ $\mathbf{t} = 4 \text{ hr.} (\mathbf{t} > 0)$	Time is NOT negative

Numbers

(4) A positive number "$\mathbf{x}$" is $\frac{15}{4}$ greater than its reciprocal "$\frac{1}{\mathbf{x}}$". Find the number "$\mathbf{x}$".

Fundamental equation: $\mathbf{x}=\frac{15}{4}+\frac{1}{\mathbf{x}}$

Step	Equation	Reason
0	$\mathbf{x}=\frac{15}{4}+\frac{1}{\mathbf{x}}$	
1	$4\mathbf{x}(\mathbf{x})=4\mathbf{x}\left(\frac{15}{4}+\frac{1}{\mathbf{x}}\right)$	Eliminate Fractions Multiply by "4x"
2	$4\mathbf{x}^2=15\mathbf{x}+4$	
3	$4\mathbf{x}^2-15\mathbf{x}-4=0$	
4	$(\mathbf{x}-4)(4\mathbf{x}+1)=0$	Factor
5	$\mathbf{x}-4=0$ ‖ $4\mathbf{x}+1=0$ $\mathbf{x}=4$ ‖ $\mathbf{x}=-1/4$	
6	$\mathbf{x}=4\ \ (\mathbf{x}>0)$	

Physics

Consider an object moving in a vertical direction only. Under certain assumptions, the **position** "$\mathbf{s} = \mathbf{s}(\mathbf{t})$" of this object at time "$\mathbf{t}$" is given by

$$\mathbf{s} = \mathbf{s}(\mathbf{t}) = \frac{1}{2}\mathbf{g}\mathbf{t}^2 + \mathbf{v}_0\mathbf{t} + \mathbf{s}_0 = -16\mathbf{t}^2 + \mathbf{v}_0\mathbf{t} + \mathbf{s}_0 \ \ (\text{ft})$$

where

$\mathbf{g} = -32$ ft/sec^2

$\mathbf{v}_0 =$ initial velocity $(\mathbf{t} = 0)$

- $\mathbf{v}_0 < 0$ means object "thrown downward"
- $\mathbf{v}_0 = 0$ means object "dropped"
- $\mathbf{v}_0 > 0$ means object "thrown upward"

$\mathbf{s}_0 =$ initial position $(\mathbf{t} = 0)$

- $\mathbf{s}_0 < 0$ is NOT an option
- $\mathbf{s}_0 = 0$ means object starts at ground
- $\mathbf{s}_0 > 0$ means object starts above ground

The **velocity** of the object is given by

$$\mathbf{v} = \mathbf{v}(\mathbf{t}) = \mathbf{v}_0 + \mathbf{g}\,\mathbf{t} = \mathbf{v}_0 - 32\mathbf{t} \ \ (\text{ft/sec})$$

- $\mathbf{v} < 0$ means object traveling "downward"
- $\mathbf{v} = 0$ means object "stopped"
- $\mathbf{v} > 0$ means object traveling "upward"

The **acceleration** is given by

$$\mathbf{a} = \mathbf{a}(\mathbf{t}) = \mathbf{g} = -32 \text{ ft/sec}^2$$

The information above comes from "Physics".

(5) The equations defining the motion of an object traveling in a vertical direction are given by

$$\mathbf{s} = \mathbf{s}(\mathbf{t}) = -16\mathbf{t}^2 + 144\mathbf{t} + 50$$

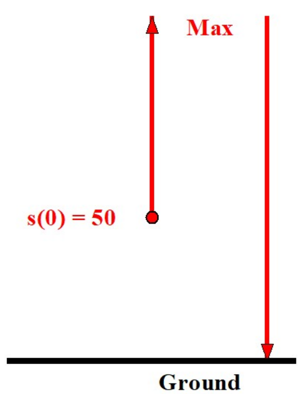
Note:

$\mathbf{v}_0 = 144$ ft/sec (object thrown upward)

$\mathbf{s}_0 = 50$ ft (object starts 50 ft above ground)

$$\mathbf{v} = \mathbf{v}(\mathbf{t}) = 144 - 32\mathbf{t}$$

$$\mathbf{a} = \mathbf{a}(\mathbf{t}) = \mathbf{g} = -32$$



a. Where is the object after 3 sec? Is it traveling upward, stopped, or downward?

$$\mathbf{s} = \mathbf{s}(3) = -16(3)^2 + 144(3) + 50 = 338 \text{ ft}$$
$$\mathbf{v} = \mathbf{v}(3) = 144 - 32(3) = 48 \text{ ft/sec} \quad (\text{upward})$$

b. Where is the object after 5 sec? Is it traveling upward/stopped/downward?

$\mathbf{s} = \mathbf{s}(5) = -16(5)^2 + 144(5) + 50 = 370 \text{ ft}$

$\mathbf{v} = \mathbf{v}(5) = 144 - 32(5) = -16 \text{ ft/sec} \quad (\text{downward})$

c. When does the object stop and where is it ?

$$\mathbf{v} = \mathbf{v}(\mathbf{t}) \overset{\text{SET}}{=} 0$$

$$144 - 32\mathbf{t} = 0$$

$$32\mathbf{t} = 144$$

$$\mathbf{t} = \frac{144}{32} = \frac{9}{2} \text{ sec}$$

$$\mathbf{s} = \mathbf{s}\left(\frac{9}{2}\right) = -16\left(\frac{9}{2}\right)^2 + 144\left(\frac{9}{2}\right) + 55 = 374 \text{ ft}$$

d. Since it was thrown upward $(\mathbf{v}_0 = 144 > 0)$, when will it return to ground? How fast is it traveling?

$$\mathbf{s} = \mathbf{s}(\mathbf{t}) \overset{\text{SET}}{=} 0$$

$$-16\mathbf{t}^2 + 144\mathbf{t} + 50 = 0$$

$$-8\mathbf{t}^2 + 72\mathbf{t} + 25 = 0$$

$$8\mathbf{t}^2 - 72\mathbf{t} - 25 = 0$$

$\mathbf{a} = 8$; $\mathbf{b} = -72$; $\mathbf{c} = -25$

$$\mathbf{t} = \frac{-\mathbf{b} \pm \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}$$

$$\mathbf{t} = \frac{-[-72] \pm \sqrt{[-72]^2 - 4[8][-25]}}{2[8]}$$

$$\mathbf{t} = \frac{72 \pm \sqrt{5184 + 800}}{16}$$

$$\mathbf{t} = \frac{72 \pm \sqrt{5984}}{16} \quad (\mathbf{t} > 0)$$

$$\mathbf{t} = \frac{72 + \sqrt{5984}}{16}$$

$$\mathbf{t} \approx 9.334 \text{ sec}$$

$\text{v}(\approx 9.334) \approx -154.713 \text{ ft/sec}$