Equations – Quadratic in Form

$$\begin{bmatrix} \text{MATH by Wilson} \\ \text{Your Personal Mathematics Trainer} \\ \text{MathByWilson.com} \end{bmatrix}$$

Quadratic Equation: $\mathbf{au}^2+\mathbf{bu}+\mathbf{c}=0$ (Standard Form) in **u** (called the universal variable)

The **solutions** are $\mathbf{u}=\dfrac{-\mathbf{b}\pm\sqrt{\mathbf{b}^2-4\mathbf{ac}}}{2\mathbf{a}}$ using the Quadratic Equation

Note: You may also use the Factoring Technique or the Completing the Square Technique.

***** **u** can be any of the following plus plenty of others in the original equation.

$\mathbf{u}=\mathbf{x}$

$\mathbf{u}=\dfrac{1}{\mathbf{x}^2}$

$\mathbf{u}=\sqrt[3]{\mathbf{x}}=\mathbf{x}^{1/3}$

$\mathbf{u}=\mathbf{x}^2$

$\mathbf{u}=\dfrac{1}{\mathbf{x}}$

$\mathbf{u}=\mathbf{x}-2$

*****

Strategy: In the original equation, we

1. set u = "something" so we get a quadratic equation in **u**:

   $$\mathbf{au}^2+\mathbf{bu}+\mathbf{c}=0$$

2. solve this equation for **u**
3. use the solutions for **u** to get the original solutions for **x**

Note: There may be more than two (2) solutions.

We'll look at equations – Quadratic in Form – to get the following equations in **u**:

*****

$\mathbf{u}^2 - 14\mathbf{u} + 45 = 0$

$\mathbf{u}^2 + 7\mathbf{u} + 12 = 0$

$\mathbf{u}^2 - 13\mathbf{u} + 36 = 0$

$6\mathbf{u}^2 - 11\mathbf{u} - 10 = 0$

*****

Equation 01: Find the solutions of the equation $\frac{1}{\mathbf{x}^4} - 14\frac{1}{\mathbf{x}^2} + 45 = 0$.

Solution:

Set $\mathbf{u} = \frac{1}{\mathbf{x}^2}$ so that

$$\frac{1}{\mathbf{x}^4} - 14\frac{1}{\mathbf{x}^2} + 45 = 0$$

$$\left(\frac{1}{\mathbf{x}^2}\right)^2 - 14\left(\frac{1}{\mathbf{x}^2}\right) + 45 = 0$$

$$\mathbf{u}^2 - 14\mathbf{u} + 45 = 0$$

Note: $\mathbf{a} = 1$; $\mathbf{b} = -14$; $\mathbf{c} = 45$

Step	Equation	Reason
0	$\mathbf{u}^2 - 14\mathbf{u} + 45 = 0$	
1	$(\mathbf{u}-9)(\mathbf{u}-5) = 0$	Factor
2	$\mathbf{u} - 9 = 0$, $\mathbf{u} = 9$ ‖ $\mathbf{u} - 5 = 0$, $\mathbf{u} = 5$	2 Linear Equations
3	$\frac{1}{\mathbf{x}^2} = 9$, $\mathbf{x}^2 = \frac{1}{9}$, $\mathbf{x} = \pm\frac{1}{3}$ ‖ $\frac{1}{\mathbf{x}^2} = 5$, $\mathbf{x}^2 = \frac{1}{5}$, $\mathbf{x} = \pm\frac{1}{\sqrt{5}} = \pm\frac{\sqrt{5}}{5} \approx \pm 0.447$	Solve for "x"

Graph of the solution set:

________ $-1/\sqrt{5}$ _ $-1/3$ ______ $\underset{0}{|}$ ______ $1/3$ __ $1/\sqrt{5}$ ____________

Equation 02: The equation $\mathbf{x}^{2/3}+7\mathbf{x}^{1/3}+12=0$ has how many solutions?
Solution:

Set $\mathbf{u}=\mathbf{x}^{1/3}$ so that

$$\mathbf{x}^{2/3}+7\mathbf{x}^{1/3}+12=0$$
$$\left(\mathbf{x}^{1/3}\right)^2+7\left(\mathbf{x}^{1/3}\right)+12=0$$
$$\mathbf{u}^2+7\mathbf{u}+12=0$$

Note: $\mathbf{a}=1$; $\mathbf{b}=7$; $\mathbf{c}=12$

Step	Equation	Reason
0	$\mathbf{u}^2+7\mathbf{u}+12=0$	
1	$(\mathbf{u}+4)(\mathbf{u}+3)=0$	Factor
2	$\mathbf{u}+4=0$ ‖ $\mathbf{u}+3=0$ $\mathbf{u}=-4$ ‖ $\mathbf{u}=-3$	2 Linear Equations
3	$\mathbf{x}^{1/3}=-4$ ‖ $\mathbf{x}^{1/3}=-3$ $\mathbf{x}=\left(\mathbf{x}^{1/3}\right)^3=(-4)^3$ ‖ $\mathbf{x}=\left(\mathbf{x}^{1/3}\right)^3=(-3)^3$ $\mathbf{x}=-64$ ‖ $\mathbf{x}=-27$	Solve For "x"

Graph of the solution set:

________ −64 ___________ −27 _________ | ____________________
0

There are two (2) solutions.

Equation 03: Find the *largest* solution of the equation $\mathbf{x}^4-13\mathbf{x}^2+36=0$.
Solution:

Set $\mathbf{u}=\mathbf{x}^2$ so that

$$\mathbf{x}^4-13\mathbf{x}^2+36=0$$
$$\left(\mathbf{x}^2\right)^2-13\left(\mathbf{x}^2\right)+36=0$$
$$\mathbf{u}^2-13\mathbf{u}+36=0$$

Note: $\mathbf{a}=1$; $\mathbf{b}=-13$; $\mathbf{c}=36$

Step	Equation	Reason
0	$\mathbf{u}^2-13\mathbf{u}+36=0$	
1	$(\mathbf{u}-4)(\mathbf{u}-9)=0$	Factor

2	$\mathbf{u}-4=0$ $\mathbf{u}=4$ ‖ $\mathbf{u}-9=0$ $\mathbf{u}=9$	
3	$\mathbf{x}^2=4$ $\mathbf{x}=\pm\sqrt{4}=\pm 2$ ‖ $\mathbf{x}^2=9$ $\mathbf{x}=\pm\sqrt{9}=\pm 3$	

Graph of the solution set:

-3 $\quad$ -2 $\quad$ 0 $\quad$ 2 $\quad$ 3

The largest solution is x = 3.

Equation 04: What is the **sum** of the solutions of the equation

$$\frac{6}{\mathbf{x}^2}-11\frac{1}{\mathbf{x}}-10=0\,?$$

Solution:

Set $\mathbf{u}=\frac{1}{\mathbf{x}}$ so that

$$\frac{6}{\mathbf{x}^2}-11\frac{1}{\mathbf{x}}-10=0$$

$$6\left(\frac{1}{\mathbf{x}}\right)^2-11\left(\frac{1}{\mathbf{x}}\right)-10=0$$

$$6\mathbf{u}^2-11\mathbf{u}-10=0$$

Note: $\mathbf{a}=6$; $\mathbf{b}=-11$; $\mathbf{c}=-10$

Step	Equation	Reason
0	$6\mathbf{u}^2-11\mathbf{u}-10=0$	
1	$(3\mathbf{u}+2)(2\mathbf{u}-5)=0$	Factor
2	$3\mathbf{u}+2=0$ $3\mathbf{u}=-2$ $\mathbf{u}=-\frac{2}{3}$ ‖ $2\mathbf{u}-5=0$ $2\mathbf{u}=5$ $\mathbf{u}=\frac{5}{2}$	Two Linear Equations

3	$\frac{1}{\mathbf{x}} = -\frac{2}{3}$ $\mathbf{x} = -\frac{3}{2}$	$\frac{1}{\mathbf{x}} = \frac{5}{2}$ $\mathbf{x} = \frac{2}{5}$	Solve For "x"

Graph of the solution set:

$$\underline{\qquad\quad} -3/2 \underline{\qquad\quad} |_0 \underline{\quad} 2/5 \underline{\qquad\qquad}$$

The sum of solutions is -11/10.