

Equations – Quadratic in Form

[MATH by Wilson
Your Personal Mathematics Trainer
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Quadratic Equation: $\mathbf{au}^2 + \mathbf{bu} + \mathbf{c} = 0$ (Standard Form) in \mathbf{u} (called the universal variable)

The **solutions** are $\mathbf{u} = \frac{-\mathbf{b} \pm \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}$ using the Quadratic Equation

Note: You may also use the Factoring Technique or the Completing the Square Technique.

***** \mathbf{u} can be any of the following plus plenty of others in the original equation.

$$\mathbf{u} = \mathbf{x}$$

$$\mathbf{u} = \frac{1}{\mathbf{x}^2}$$

$$\mathbf{u} = \sqrt[3]{\mathbf{x}} = \mathbf{x}^{1/3}$$

$$\mathbf{u} = \mathbf{x}^2$$

$$\mathbf{u} = \frac{1}{\mathbf{x}}$$

$$\mathbf{u} = \mathbf{x} - 2$$

Strategy: In the original equation, we

1. set $\mathbf{u} =$ “something” so we get a quadratic equation in \mathbf{u} :

$$\mathbf{au}^2 + \mathbf{bu} + \mathbf{c} = 0$$

2. solve this equation for \mathbf{u}
3. use the solutions for \mathbf{u} to get the original solutions for \mathbf{x}

Note: There may be more than two (2) solutions.

We’ll look at equations – Quadratic in Form – to get the following equations in \mathbf{u} :

$$u^2 - 14u + 45 = 0$$

$$u^2 + 7u + 12 = 0$$

$$u^2 - 13u + 36 = 0$$

$$6u^2 - 11u - 10 = 0$$

Equation 01: Find the solutions of the equation $\frac{1}{x^4} - 14\frac{1}{x^2} + 45 = 0$.

Solution:

Set $u = \frac{1}{x^2}$ so that

$$\frac{1}{x^4} - 14\frac{1}{x^2} + 45 = 0$$

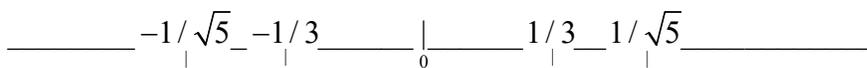
$$\left(\frac{1}{x^2}\right)^2 - 14\left(\frac{1}{x^2}\right) + 45 = 0$$

$$u^2 - 14u + 45 = 0$$

Note: $a = 1$; $b = -14$; $c = 45$

Step	Equation	Reason
0	$u^2 - 14u + 45 = 0$	
1	$(u - 9)(u - 5) = 0$	Factor
2	$u - 9 = 0 \quad \quad u - 5 = 0$ $u = 9 \quad \quad u = 5$	2 Linear Equations
3	$\frac{1}{x^2} = 9 \quad \quad \frac{1}{x^2} = 5$ $x^2 = \frac{1}{9} \quad \quad x^2 = \frac{1}{5}$ $x = \pm \frac{1}{3} \quad \quad x = \pm \frac{1}{\sqrt{5}} = \pm \frac{\sqrt{5}}{5} \approx \pm 0.447$	Solve for "x"

Graph of the solution set:



Equation 02: The equation $x^{2/3} + 7x^{1/3} + 12 = 0$ has how many solutions?

Solution:

Set $u = x^{1/3}$ so that

$$x^{2/3} + 7x^{1/3} + 12 = 0$$

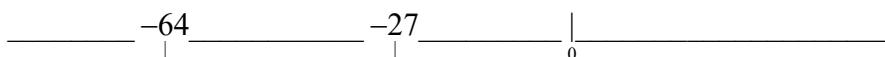
$$(x^{1/3})^2 + 7(x^{1/3}) + 12 = 0$$

$$u^2 + 7u + 12 = 0$$

Note: $a = 1$; $b = 7$; $c = 12$

Step	Equation	Reason
0	$u^2 + 7u + 12 = 0$	
1	$(u + 4)(u + 3) = 0$	Factor
2	$u + 4 = 0 \quad \quad u + 3 = 0$ $u = -4 \quad \quad u = -3$	2 Linear Equations
3	$x^{1/3} = -4 \quad \quad x^{1/3} = -3$ $x = (x^{1/3})^3 = (-4)^3 \quad \quad x = (x^{1/3})^3 = (-3)^3$ $x = -64 \quad \quad x = -27$	Solve For "x"

Graph of the solution set:



There are two (2) solutions.

Equation 03: Find the *largest* solution of the equation $x^4 - 13x^2 + 36 = 0$.

Solution:

Set $u = x^2$ so that

$$x^4 - 13x^2 + 36 = 0$$

$$(x^2)^2 - 13(x^2) + 36 = 0$$

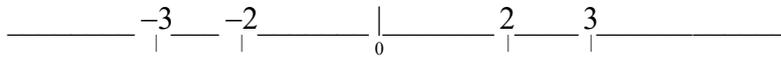
$$u^2 - 13u + 36 = 0$$

Note: $a = 1$; $b = -13$; $c = 36$

Step	Equation	Reason
0	$u^2 - 13u + 36 = 0$	
1	$(u - 4)(u - 9) = 0$	Factor

2	$u - 4 = 0$ $u = 4$	$u - 9 = 0$ $u = 9$	
3	$x^2 = 4$ $x = \pm\sqrt{4} = \pm 2$	$x^2 = 9$ $x = \pm\sqrt{9} = \pm 3$	

Graph of the solution set:



The largest solution is $x = 3$.

Equation 04: What is the **sum** of the solutions of the equation

$$\frac{6}{x^2} - 11\frac{1}{x} - 10 = 0?$$

Solution:

Set $u = \frac{1}{x}$ so that

$$\frac{6}{x^2} - 11\frac{1}{x} - 10 = 0$$

$$6\left(\frac{1}{x}\right)^2 - 11\left(\frac{1}{x}\right) - 10 = 0$$

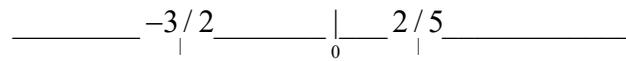
$$6u^2 - 11u - 10 = 0$$

Note: $a = 6$; $b = -11$; $c = -10$

Step	Equation	Reason
0	$6u^2 - 11u - 10 = 0$	
1	$(3u + 2)(2u - 5) = 0$	Factor
2	$3u + 2 = 0$ $2u - 5 = 0$ $3u = -2$ $2u = 5$ $u = -\frac{2}{3}$ $u = \frac{5}{2}$	Two Linear Equations

3	$\frac{1}{x} = -\frac{2}{3}$ $x = -\frac{3}{2}$	$\frac{1}{x} = \frac{5}{2}$ $x = \frac{2}{5}$	Solve For "x"
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Graph of the solution set:



The sum of solutions is $-1\frac{1}{10}$.