Equations – Radical [Roots]

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The **basic idea** is to trade an equation with one or more radicals - $\left(\sqrt[n]{\text{Expression}}\right)$ - for an equation that we have already studied that does NOT have radicals:

Original Equation: Contains Radicals

TRADE

New Equation: Does NOT Contain Radicals

Note that $\left(\sqrt[n]{\text{Expression}}\right)^n = \left[\left(\text{Expression}\right)^{1/n}\right]^n = \text{Expression}$ is key to our success!

Warning: The theory tells me to tell you that a solution of the **New Equation** does NOT have to be a solution of the **Original Equation.** Therefore, it is **mandatory** to check "potential" solutions to ensure that they are "actual" solutions.

Note: We should *always* graph our solutions, if any, on the number line.

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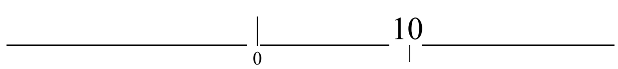
Equation 01: Solve for x: $\sqrt{2\mathbf{x}+5}=\dfrac{\mathbf{x}}{2}$

Solution:

Step	Equation	Reason
0	$\sqrt{2\mathbf{x}+5}=\dfrac{\mathbf{x}}{2}$	
1	$\left(\sqrt{2\mathbf{x}+5}\right)^2=\left(\dfrac{\mathbf{x}}{2}\right)^2$	Eliminate Radical
2	$2\mathbf{x}+5=\dfrac{\mathbf{x}^2}{4}$	
3	$8\mathbf{x}+20=\mathbf{x}^2$	Quadratic Equation
4	$\mathbf{x}^2-8\mathbf{x}-20=0$	
5	$(\mathbf{x}+2)(\mathbf{x}-10)=0$	Factor
6	$\mathbf{x}+2=0$ ‖ $\mathbf{x}-10=0$ $\mathbf{x}=-2$ ‖ $\mathbf{x}=10$	
7	$\mathbf{x}=-2$: Is NOT a solution $\sqrt{2[-2]+5}\overset{?}{=}\dfrac{[-2]}{2}$ $1\neq-1$	
8	$\mathbf{x}=10$: Is a solution $\sqrt{2[10]+5}\overset{?}{=}\dfrac{[10]}{2}$ $5=5$	

Graph of the solution set:



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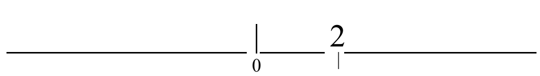
Equation 02: Solve for x: $\sqrt{6-\mathbf{x}}+\sqrt{\mathbf{x}+2}=4$ (This is a long problem because there are two radicals to eliminate **and** they must be eliminated correctly!
Solution:

Step	Equation	Reason
0	$\sqrt{6-\mathbf{x}}+\sqrt{\mathbf{x}+2}=4$	
1	$\sqrt{6-\mathbf{x}}=4-\sqrt{\mathbf{x}+2}$	Isolate one radical
2	$\left(\sqrt{6-\mathbf{x}}\right)^2=\left(4-\sqrt{\mathbf{x}+2}\right)^2$	Eliminate 1st Radical
3	$6-\mathbf{x}=16-8\sqrt{\mathbf{x}+2}+(\mathbf{x}+2)$	
4	$6-\mathbf{x}=\mathbf{x}+18-8\sqrt{\mathbf{x}+2}$	
5	$-2\mathbf{x}-12=-8\sqrt{\mathbf{x}+2}$	
6	$\mathbf{x}+6=4\sqrt{\mathbf{x}+2}$	Divide by "-2"
7	$(\mathbf{x}+6)^2=\left(4\sqrt{\mathbf{x}+2}\right)^2$	Eliminate 2nd Radical
8	$\mathbf{x}^2+12\mathbf{x}+36=16(\mathbf{x}+2)=16\mathbf{x}+32$	
9	$\mathbf{x}^2-4\mathbf{x}+4=0$	Quadratic Equation
10	$(\mathbf{x}-2)^2=0$	
11	$\mathbf{x}-2=0$ ‖ $\mathbf{x}-2=0$ $\mathbf{x}=2$ ‖ $\mathbf{x}=2$	
12	$\mathbf{x}=2$: Is a solution $\sqrt{6-[2]}+\sqrt{[2]+2}\stackrel{?}{=}4$ $2+2\stackrel{?}{=}4$ $4=4$	

Graph of the solution set:



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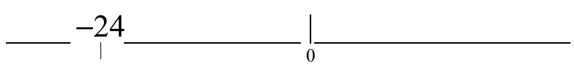
Equation 03: Solve for x: $\sqrt[3]{3-\mathbf{x}}=3$
Solution:

Step	Equation	Reason
0	$\sqrt[3]{3-\mathbf{x}}=3$	
1	$\left(\sqrt[3]{3-\mathbf{x}}\right)^3=(3)^3$	Eliminate Radical
2	$3-\mathbf{x}=27$	Linear Equation
3	$-\mathbf{x}=24 \Rightarrow \mathbf{x}=-24$	
7	$\mathbf{x}=-24$: Is a solution $\sqrt[3]{3-[-24]}\overset{?}{=}3$ $\sqrt[3]{27}\overset{?}{=}3$ $3=3$	

Graph of the solution set:



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Equation 04: Solve for x: $(\mathbf{x}+2)^{2/3} = 25$

Solution:

Step	Equation	Reason
0	$(\mathbf{x}+2)^{2/3} = 25$ $\left[(\mathbf{x}+2)^{1/3}\right]^2 = \mathbf{u}^2 = 25$; $\mathbf{u} = (\mathbf{x}+2)^{1/3}$ $(\mathbf{x}+2)^{1/3} = \pm\sqrt{25} = \pm 5$	$\mathbf{u}^2 = 25$ Implies two (2) solutions
1	$(\mathbf{x}+2)^{1/3} = -5$ ‖ $(\mathbf{x}+2)^{1/3} = +5$ $\mathbf{x}+2 = -125$ ‖ $\mathbf{x}+2 = 125$ $\mathbf{x} = -127$ ‖ $\mathbf{x} = 123$	
2	$\mathbf{x} = -127$: Is a solution $(-127+2)^{2/3} = \left[(-125)^{1/3}\right]^2 = 25$	
3	$\mathbf{x} = 123$: Is a solution $(123+2)^{2/3} = \left[(125)^{1/3}\right]^2 = 25$	

Graph of the solution set:

____ −127____________|0________ 123_________

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