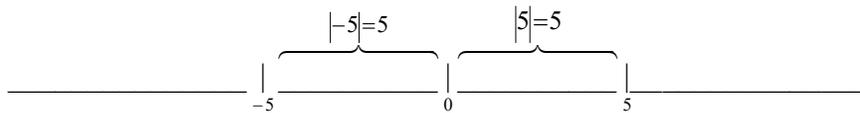


Equations – Absolute Value

[MATH by Wilson
Your Personal Mathematics Trainer
MathByWilson.com]

Definition: The **absolute value** of a number “x”, written $|x|$, refers to the size (magnitude, distance from 0, ...). It will always be either 0 ($|0| = 0$) or a positive number ($|-5| = 5 = |5|$).



Note: $|x| \geq 0$ & $|x| \neq 0$

INFORMALLY: If $x < 0$, to find the absolute value of x, just throw away the minus sign: $|-5| = 5$. If $x > 0$, the absolute value of x is just x: $|5| = 5$.

FORMALLY:

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ x & \text{if } x > 0 \end{cases}$$

Using the “formal definition”, we have

$$\begin{aligned} |-5| &= -(-5) = 5 \\ |0| &= 0 \\ |5| &= 5 \end{aligned}$$

We obtain the same results if we do things “informally”. For now, just throw away the minus sign if there is one: $|-5| = 5$

KEY: The solution of $|u| = b$; $b \geq 0$ is the *union* of the solutions of

- a. $u = -b$
- b. $u = +b$

TRADE $|u| = b$; $b \geq 0$ for $u = -b$ AND $u = +b$

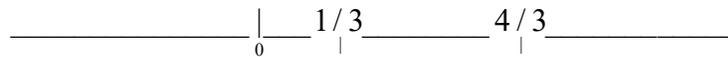
Note: An absolute value equation trades for two (2) other equations!

Example 01: Solve for x: $|5 - 6x| = 3$

Solution:

Step	Equation	Reason
0	$ 5 - 6x = 3$	
1	$\begin{array}{l} 5 - 6x = -3 \\ 6x = 8 \\ x = \frac{8}{6} = \frac{4}{3} \end{array} \quad \Bigg \quad \begin{array}{l} 5 - 6x = 3 \\ 6x = 2 \\ x = \frac{1}{3} \end{array}$	Trade for 2 Linear Equations

Graph of the solution set:



Example 02: Solve for x: $|x^2 + 2x| = 2$

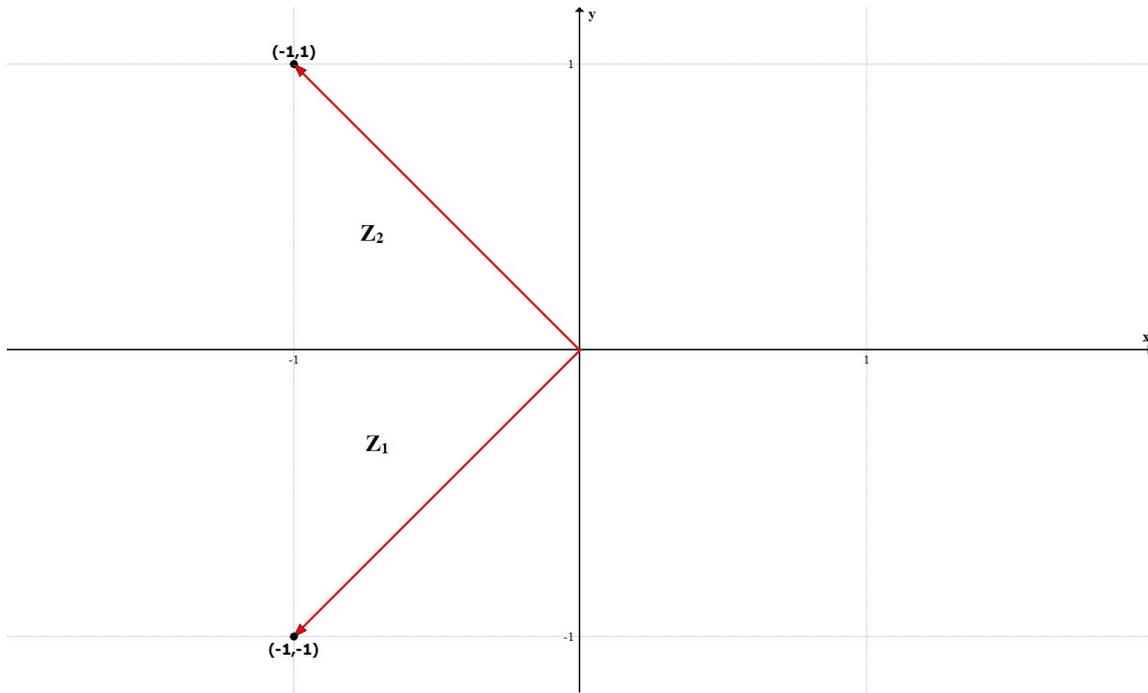
Solution:

Step	Equation	Reason
0	$ x^2 + 2x = 2$	
1	$\begin{array}{l} x^2 + 2x = -2 \\ x^2 + 2x + 2 = 0 \\ x = \frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2} \\ = \frac{-2 \pm \sqrt{-4}}{2} \\ = \frac{-2 \pm 2i}{2} \\ = -1 \pm i \end{array} \quad \Bigg \quad \begin{array}{l} x^2 + 2x = 2 \\ x^2 + 2x - 2 = 0 \\ x = \frac{2 \pm \sqrt{2^2 - 4(1)(-2)}}{2} \\ = \frac{2 \pm \sqrt{12}}{2} \\ = \frac{2 \pm 2\sqrt{3}}{2} \\ = 1 \pm \sqrt{3} \end{array}$	Trade for 2 Quadratic Equations

Graphs: Four (4) Solutions

$$\frac{1-\sqrt{3}}{0} \quad | \quad \frac{1+\sqrt{3}}{0}$$

AND

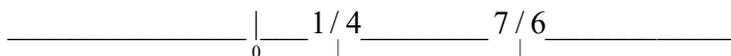


Example 03: Solve for x: $|x - 3| = |4 - 5x|$

Solution:

Step	Equation	Reason		
0	$ x - 3 = 4 - 5x $ $\pm(x - 3) = \pm(4 - 5x) \Rightarrow (x - 3) = \pm(4 - 5x)$			
1	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="border-right: 1px solid black; padding: 5px;"> $x - 3 = -(4 - 5x)$ $x - 3 = -4 + 5x$ $4 - 3 = 5x - x$ $1 = 4x$ $x = \frac{1}{4}$ </td> <td style="padding: 5px;"> $x - 3 = +(4 - 5x)$ $x - 3 = 4 - 5x$ $x + 5x = 4 + 3$ $6x = 7$ $x = \frac{7}{6}$ </td> </tr> </table>	$x - 3 = -(4 - 5x)$ $x - 3 = -4 + 5x$ $4 - 3 = 5x - x$ $1 = 4x$ $x = \frac{1}{4}$	$x - 3 = +(4 - 5x)$ $x - 3 = 4 - 5x$ $x + 5x = 4 + 3$ $6x = 7$ $x = \frac{7}{6}$	Trade for 2 Linear Equations
$x - 3 = -(4 - 5x)$ $x - 3 = -4 + 5x$ $4 - 3 = 5x - x$ $1 = 4x$ $x = \frac{1}{4}$	$x - 3 = +(4 - 5x)$ $x - 3 = 4 - 5x$ $x + 5x = 4 + 3$ $6x = 7$ $x = \frac{7}{6}$			

Graph of the solution set:



For Your Information (FYI)

FYI 01: Consider $x^2 + 2x + 2 = 0$

$$x = -1 - i \text{ \& } x = -1 + i$$

$$x + 1 + i = 0 \text{ \& } x + 1 - i = 0$$

$$(x + 1 + i)(x + 1 - i) = 0$$

$$x^2 + x - xi + x + 1 - i + xi + i - i^2 = 0$$

$$x^2 + 2x + 2 = 0$$

Hence $x^2 + 2x + 2$ factors as $(x + 1 + i)(x + 1 - i)$ **UGLY Factors!**

Substituting $x = -1 + i$ into $x^2 + 2x + 2$ yields

$$\begin{aligned} x^2 + 2x + 2 &= (-1 + i)^2 + 2(-1 + i) + 2 \\ &= 1 - 2i + i^2 - 2 + 2i + 2 \\ &= 0 \end{aligned}$$

This calculation verifies that $x = -1 + i$ is a solution of $x^2 + 2x + 2 = 0$

Your Turn: Show that $x = -1 - i$ is also a solution of $x^2 + 2x + 2 = 0$

FYI 02: Consider $x^2 + 2x - 2 = 0$

$$x = -1 - \sqrt{3} \text{ \& } x = -1 + \sqrt{3}$$

$$x + 1 + \sqrt{3} = 0 \text{ \& } x + 1 - \sqrt{3} = 0$$

$$(x + 1 + \sqrt{3})(x + 1 - \sqrt{3}) = 0$$

$$x^2 + x - \sqrt{3}x + x + 1 - \sqrt{3} + \sqrt{3}x + \sqrt{3} - (\sqrt{3})^2 = 0$$

$$x^2 + 2x - 2 = 0$$

Hence $x^2 + 2x - 2$ factors as $(x + 1 + \sqrt{3})(x + 1 - \sqrt{3})$ **UGLY Factors!**

Substituting $x = -1 + \sqrt{3}$ into $x^2 + 2x - 2$ yields

$$\begin{aligned}x^2 + 2x - 2 &= (-1 + \sqrt{3})^2 + 2(-1 + \sqrt{3}) - 2 \\&= 1 - 2\sqrt{3} + (\sqrt{3})^2 - 2 + 2\sqrt{3} - 2 \\&= 0\end{aligned}$$

This calculation verifies that $x = -1 + \sqrt{3}$ is a solution of $x^2 + 2x - 2 = 0$

Your Turn: Show that $x = -1 - \sqrt{3}$ is also a solution of $x^2 + 2x - 2 = 0$