# Inequalities
# Linear

MATH by Wilson
Your Personal Mathematics Trainer
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**We consider two (2) types of linear inequalities:**

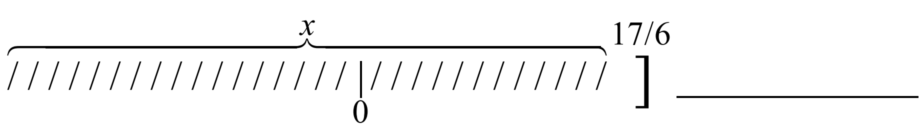
**1. One Inequality Symbol:** $\{\le ; < ; \ge ; >\}$

**Example 01: Solve for x:** $4-3(2+\mathbf{x}) \le \mathbf{x}+5(3-2\mathbf{x})$
**Solution:**

| Step | Inequality | Reason |
|---|---|---|
| **0** | $4-3(2+\mathbf{x}) \le \mathbf{x}+5(3-2\mathbf{x})$ | |
| **1** | $4-6-3\mathbf{x} \le \mathbf{x}+15-10\mathbf{x}$ | |
| **2** | $-2-3\mathbf{x} \le -9\mathbf{x}+15$ | |
| **3** | $9\mathbf{x}-3\mathbf{x} \le 15+2$ | |
| **4** | $6\mathbf{x} \le 17$ | |
| **5** | $\mathbf{x} \le \frac{17}{6}$ | |

Solution in interval notation: $\left(-\infty, \frac{17}{6}\right]$

Graph of the solution set:



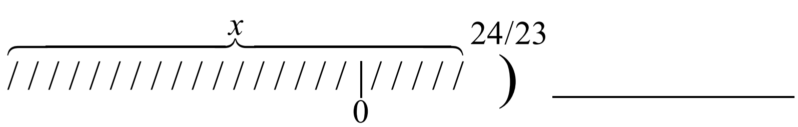
**Example 02: Solve for x:** $3-\frac{7}{3}\mathbf{x}>4\left(\frac{3}{8}\mathbf{x}+1\right)-5$

**Solution:**

| Step | Inequality | Reason |
|---|---|---|
| **0** | $3-\frac{7}{3}\mathbf{x}>4\left(\frac{3}{8}\mathbf{x}+1\right)-5$ | |
| **1** | $3-\frac{7}{3}\mathbf{x}>4\left(\frac{3}{8}\mathbf{x}+1\right)-5=\frac{3\mathbf{x}}{2}-1$ | |
| **2** | $\frac{3}{1}-\frac{7}{3}\mathbf{x}>\frac{3\mathbf{x}}{2}-\frac{1}{1}$ | All Fractions |
| **3** | $6\left(\frac{3}{1}-\frac{7}{3}\mathbf{x}\right)>6\left(\frac{3\mathbf{x}}{2}-\frac{1}{1}\right)$ | Multiply by Common Denominator " 6" |
| **4** | $18-14\mathbf{x}>9\mathbf{x}-6$ | |
| **5** | $-14\mathbf{x}-9\mathbf{x}>-6-18$ | |
| **6** | $-23\mathbf{x}>-24$ | |
| **7** | $\mathbf{x}<\frac{-24}{-23}=\frac{24}{23}$ | Note switch in direction |

Solution in interval notation: $\left(-\infty,\frac{24}{23}\right]$

Graph of the solution set:




**2. Two Inequality Symbols:** $\{\le;<;\ge;>\}$ **- inequality symbols pointing the same direction**

**Example 03: Solve for x:** $-4<1-3\mathbf{x}\le 4$
**Solution:**

**Note: The solution of** $-4<1-3\mathbf{x}\le 4$ **is actually the solution of two (2) inequalities**

$-4<1-3\mathbf{x}$ and $1-3\mathbf{x}\le 4$

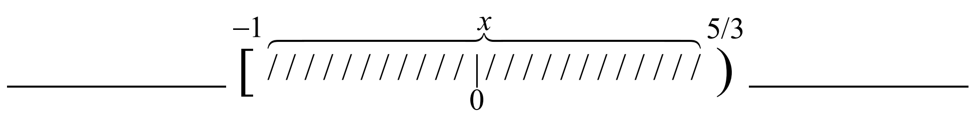
 

**but they can be written and solved as $-4 < 1 - 3\mathbf{x} \le 4$ since the inequality symbols are pointing in the same direction.**

| Step | Inequality | Reason |
|---|---|---|
| **0** | $-4 < 1 - 3\mathbf{x} \le 4$ | **Goal: Isolate "x"** |
| **1** | $-4 - 1 < -3\mathbf{x} \le 4 - 1$ | |
| **2** | $-5 < -3\mathbf{x} \le 3$ | |
| **3** | $\frac{-5}{-3} > \mathbf{x} \ge \frac{3}{-3}$ | **Divide by a negative**<br>**Change direction**<br>**of inequality symbol** |
| **4** | $\frac{5}{3} > \mathbf{x} \ge -1$<br>$-1 \le \mathbf{x} < \frac{5}{3}$ | |

Solution set in interval notation: $\left[-1, \frac{5}{3}\right)$

Graph of the solution set:



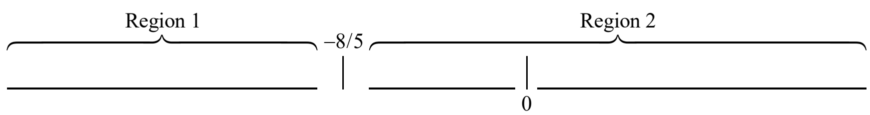

We now show an additional way to solve a linear inequality when a solution exists. Consider

$$5 - 2(\mathbf{x} + 3) \ge 3\mathbf{x} + 7$$

First, we solve the corresponding linear equation

$$5 - 2(\mathbf{x} + 3) = 3\mathbf{x} + 7$$
$$5 - 2\mathbf{x} - 6 = 3\mathbf{x} + 7$$
$$-1 - 7 = 3\mathbf{x} + 2\mathbf{x}$$
$$-8 = 5\mathbf{x}$$
$$\mathbf{x} = -\frac{8}{5}$$

Since the inequality is $" \geq "$**,** the number $\mathbf{x} = -\frac{8}{5}$ will be a solution. Now the number $\mathbf{x} = -\frac{8}{5}$**,** called a **boundary point**, divides the number line into two (2) regions:




One region, including $\mathbf{x} = -\frac{8}{5}$, will be the solution set; the other region will NOT. Now to find out, just pick a point in each region and determine which point satisfies the original inequality:

1. Choose, say x = -3:

   $$5 - 2\left(\boxed{-3} + 3\right) \overset{?}{\geq} 3 * \boxed{-3}$$

   $$5 \overset{?}{\geq} -9$$

   **TRUE**!

2. Choose, say x = 0:

   $$5 - 2\left(\boxed{0} + 3\right) \overset{?}{\geq} 3 * \boxed{0}$$

   $$-1 \overset{?}{\geq} 0$$

   **FALSE**!

The solution set is $\left(-\infty, -\dfrac{8}{5}\right]$ **OR** $\mathbf{x} \le -\dfrac{8}{5}$