Inequalities
Quadratic

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Recall that we can solve *linear inequalities* by finding the solution of the corresponding equation (called a **Boundary Point**) and testing the two (2) regions it defines by selecting any point in each region and seeing if the original inequality is true (or false). This is called a **Test Point Method** and will be modified to solve quadratic Inequalities:

Procedure (Test Point Method):

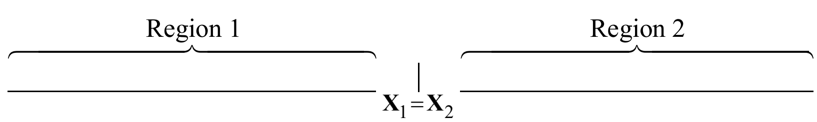
1. **With the quadratic inequality in standard form**

$$\mathbf{ax^2+bx+c}\left\{\begin{matrix} < \\ \le \\ > \\ \ge \end{matrix}\right\} 0$$

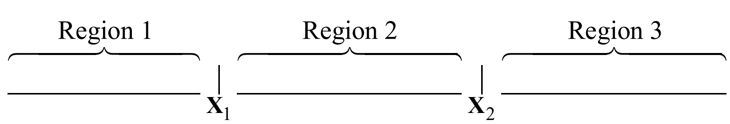
find the solutions $\mathbf{x_1}$; $\mathbf{x_2}$ **of** $\mathbf{ax^2+bx+c}=0$. **The solutions divide the horizontal number line into regions:**

   a. **One (1) region if the solutions are complex numbers.**

   b. **Two (2) regions if the solutions are the same real number, that is** $\mathbf{x_1=x_2}$.



   c. **Three (3) regions if the solutions differ, that is,** $\mathbf{x_1\neq x_2}$



2. **If x_1 or x_2 satisfies the original inequality, the numbers are part of the solution set. If not, they are not!**
3. **Pick any valve inside each region. If the value satisfies the original inequality, then all of the numbers in the region are part of the solution set. If not, they are not!**
4. **Write the solution set using interval notation and graph the solution set.**

Example 01: Solve for x: $\mathbf{x}^2 < 4$
Solution:

Step	Inequality	Reason
0	$\mathbf{x}^2 < 4$	
1	Determine Boundary Points: $\mathbf{x}^2 = 4$ $\mathbf{x} = \pm 2$; Boundary Points NOT in solution set!	
2	Check Intervals 1. $(-\infty,-2)$: Test Point $\mathbf{x}=-10; (-10)^2 \overset{?}{<} 4 \Rightarrow 100 \overset{?}{<} 4 \Rightarrow$ **False** 2. $(-2,2)$: Test Point $\mathbf{x}=0; (0)^2 \overset{?}{<} 4 \Rightarrow 0 \overset{?}{<} 4 \Rightarrow$ **True** ; in the solution set 3. $(2,+\infty)$: Test Point $\mathbf{x}=10; (10)^2 \overset{?}{<} 4 \Rightarrow 100 \overset{?}{<} 4 \Rightarrow$ **False**	
3	Solution Set: $(-2,2)$	

Graph of the solution set:

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                 -2                      2
 ____________ ( //////////|////////// ) ______________
                          0
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Example 02: Solve for x: $(\mathbf{x}-3)(\mathbf{x}+1) \geq 21$
Solution:

Step	Inequality	Reason
0	$(\mathbf{x}-3)(\mathbf{x}+1) \geq 21$	
1	$\mathbf{x}^2 - 2\mathbf{x} - 3 \geq 21$	
2	$\mathbf{x}^2 - 2\mathbf{x} - 24 \geq 0$	Standard Quadratic Form

3	$(\mathbf{x}-6)(\mathbf{x}+4)\ge 0$	Factor
4	Determine Boundary Points: $(\mathbf{x}+4)(\mathbf{x}-6)=0$ $\mathbf{x}=-4$; $\mathbf{x}=6$	
5	Check Boundary Points: 1. $\mathbf{x}=-4:([-4]-3)([-4]+1)\overset{?}{\ge}21$ $(-7)(-3)\overset{?}{\ge}21$ $21\ge 21$ True ; -4 is in the solution set 2. $\mathbf{x}=6:([6]-3)([6]+1)\overset{?}{\ge}21$ $(3)(7)\overset{?}{\ge}21$ $21\ge 21$ True ; 6 is in the solution set	
6	Check Intervals: 1. $(-\infty,-4)$: Test Point $\mathbf{x}=-6;([-6]-3)([-6]+1)\overset{?}{\ge}21$ $(-9)(-5)\overset{?}{\ge}21$ $45\ge 21$ True ; $(-\infty,-4)$ is in the solution set 2. $(-4,6)$: Test Point $\mathbf{x}=0;([0]-3)([0]+1)\overset{?}{\ge}21$ $(-3)(1)\overset{?}{\ge}21$ $-3\ge 21$ False 3. $(6,+\infty)$: Test Point $\mathbf{x}=10;([10]-3)([10]+1)\overset{?}{\ge}21$ $(7)(11)\overset{?}{\ge}21$ $77\ge 21$ True ; $(6,+\infty)$ is in the solution set	
7	Solution Set: $(-\infty,-4]\cup[6,+\infty)$	

Graph of the solution set: