Inequalities – Rational

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The *standard form* for a **rational inequality** is

$$\frac{\textbf{Expression}\,1}{\textbf{Expression}\,2}\begin{Bmatrix} < \\ \le \\ > \\ \ge \end{Bmatrix}0$$

A rational inequality seldom comes in this form so we must first put it in the standard form *before* we proceed:

Test Point Method:

1. Put the original rational inequality in standard form:

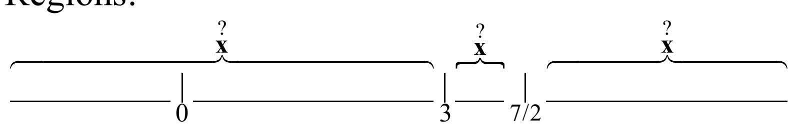
   $$\frac{\textbf{Expression}\,1}{\textbf{Expression}\,2}\begin{Bmatrix} < \\ \le \\ > \\ \ge \end{Bmatrix}0$$

2. To obtain the **boundary points**

   a. Set $\textbf{Expression}\,1 = 0$ (These values *will* be solutions of the original rational inequality when the inequality symbol is $"\le"$ **OR** $"\ge"$.
   b. Set $\textbf{Expression}\,2 = 0$ (These values *will* **NEVER** be solutions since they make the denomination zero: "0")

3. The boundary points divide the number line into at least one (1) region.
4. Select a number in each region. If it satisfies the original rational inequality, that region is part of the solution set. If not, it's NOT.
5. Write the solution set in *interval notation* and graph it on the number line.

Example 01: Solve for x: $\frac{2}{\mathbf{x}-3}<4$

Step	Inequality	Reason
0	$\frac{2}{\mathbf{x}-3}<4$	$\mathbf{x}\neq 3$
1	$\frac{2}{\mathbf{x}-3}-\frac{4}{1}<0$	Fractional Form
2	$\frac{2}{\mathbf{x}-3}-\frac{4(\mathbf{x}-3)}{1(\mathbf{x}-3)}<0$	Common Denominator
3	$\frac{2-4(\mathbf{x}-3)}{\mathbf{x}-3}<0$	
4	$\frac{2-4\mathbf{x}+12}{\mathbf{x}-3}<0$	
5	$\frac{14-4\mathbf{x}}{\mathbf{x}-3}<0$	Standard Form
6	Determine Boundary Points: 1. $\frac{14-4\mathbf{x}}{\mathbf{x}-3}=0$ $14-4\mathbf{x}=0$ $\mathbf{x}=\frac{14}{4}=\frac{7}{2}$ (NOT a solution) 2. $\frac{14-4\mathbf{x}}{\mathbf{x}-3}=$ Undefined $\mathbf{x}-3=0$ $\mathbf{x}=3$ (NEVER a solution)	
7	Regions: 	
8	Test the Regions: 1. $(-\infty,3):\mathbf{x}=0;\frac{2}{\boxed{0}-3}\overset{?}{<}4$ $-\frac{2}{3}\overset{?}{<}4$ **True**	

	2. $\left(3,\frac{7}{2}\right): \mathbf{x}=\frac{13}{4}; \frac{2}{\boxed{13/4}-3}\overset{?}{<}4$ $\frac{2/1}{1/4}\overset{?}{<}4$ $8\overset{?}{<}4$ **False** 3. $\left(\frac{7}{2};+\infty\right): \mathbf{x}=10\,; \frac{2}{\boxed{10}-3}\overset{?}{<}4$ $\frac{2}{7}\overset{?}{<}4$ **True**	
9.	Solution set: $(-\infty,3)\cup\left(\frac{7}{2},+\infty\right)$	

Graph of solution set:

Equation 02: Solve for x: $\frac{2\mathbf{x}-3}{\mathbf{x}+4}\le 5$

Solution:

Step	Inequality	Reason
0	$\frac{2\mathbf{x}-3}{\mathbf{x}+4}\le 5$	$\mathbf{x}\neq 4$
1	$\frac{2\mathbf{x}-3}{\mathbf{x}+4}-5\le 0$	
2	$\frac{2\mathbf{x}-3}{\mathbf{x}+4}-\frac{5(\mathbf{x}+4)}{(\mathbf{x}+4)}\le 0$	
3	$\frac{(2\mathbf{x}-3)-5(\mathbf{x}+4)}{\mathbf{x}+4}\le 0$	
4	$\frac{-3\mathbf{x}-23}{\mathbf{x}+4}\le 0$ OR $-\frac{3\mathbf{x}+23}{\mathbf{x}+4}\le 0$ OR $\frac{3\mathbf{x}+23}{\mathbf{x}+4}\ge 0$	

5	Determine Boundary Points: 1. $\frac{3\mathbf{x}+23}{\mathbf{x}+4}=0$ $3\mathbf{x}+23=0$ $\mathbf{x}=-\frac{23}{3}$ 2. $\frac{3\mathbf{x}+23}{\mathbf{x}+4}=$ Undefined $\mathbf{x}+4=0$ $\mathbf{x}=-4$	
6	Check Boundary Points: 1. $\mathbf{x}=-\frac{23}{3}:\frac{2\left[-\frac{23}{3}\right]-3}{\left[-\frac{23}{3}\right]+4}\overset{?}{\le}5$ $\frac{2\left[-\frac{23}{3}\right]-3}{\left[-\frac{23}{3}\right]+4}\cdot\frac{3}{3}\overset{?}{\le}5$ $\frac{-46-9}{-23+12}\overset{?}{\le}5$ $\frac{-55}{-11}\overset{?}{\le}5$ $5\overset{?}{\le}5$ True 2. $\mathbf{x}=-4:\frac{2[-4]-3}{[-4]+4}\overset{?}{\le}5$ Division by Zero: False	

7	Check Intervals: 1. $\left(-\infty,-\frac{23}{3}\right)$: Test Point $\mathbf{x}=-10$; $\frac{2[-10]-3}{[-10]+4}\overset{?}{\le}5$ $\frac{-23}{-6}\overset{?}{\le}5$ $\frac{23}{6}\overset{?}{\le}5$ True 2. $\left(-\frac{23}{3},-4\right)$: Test Point $\mathbf{x}=-5$; $\frac{2[-5]-3}{[-5]+4}\overset{?}{\le}5$ $\frac{-13}{-1}\overset{?}{\le}5$ $13\overset{?}{\le}5$ False 3. $(-4,+\infty)$: Test Point $\mathbf{x}=10$; $\frac{2[10]-3}{[10]+4}\overset{?}{\le}5$ $\frac{17}{14}\overset{?}{\le}5$ True	
8	Solution: $\left(-\infty,-\frac{23}{3}\right]\cup(-4,+\infty)$	

Graph of the solution set:

Question 03: Solve for x: $\frac{3\mathbf{x}}{16-\mathbf{x}^2}\ge\frac{1}{2}$

Solution:

Step	Inequality	Reason
0	$\frac{3\mathbf{x}}{16-\mathbf{x}^2}\ge\frac{1}{2}$	$\mathbf{x}\ne\pm4$
1	$\frac{3\mathbf{x}}{16-\mathbf{x}^2}-\frac{1}{2}\ge 0$	
2	$\frac{2(3\mathbf{x})}{2(16-\mathbf{x}^2)}-\frac{1(16-\mathbf{x}^2)}{2(16-\mathbf{x}^2)}\ge 0$	

3	$-2\left[\frac{6\mathbf{x}-16+\mathbf{x}^2}{2\left(16-\mathbf{x}^2\right)}\right]\ge -2[0]$	
4	$\frac{\mathbf{x}^2+6\mathbf{x}-16}{\mathbf{x}^2-16}\le 0$	
5	Determine Boundary Points: 1. $\frac{\mathbf{x}^2+6\mathbf{x}-16}{\mathbf{x}^2-16}=0$ $\mathbf{x}^2+6\mathbf{x}-16=(\mathbf{x}+8)(\mathbf{x}-2)=0$ $\mathbf{x}=-8$, 2 in solution set 2. $\frac{\mathbf{x}^2+6\mathbf{x}-16}{\mathbf{x}^2-16}=$ Undefined $\mathbf{x}=\pm 4$ NEVER in solution set	
6	Check Intervals: 1. $(-\infty,-8)$: Test Point $\mathbf{x}=-10$; $\frac{3*\boxed{-10}}{16-\boxed{-10}^2}=\frac{-30}{-84}=\frac{5}{14}\overset{?}{\ge}\frac{1}{2}$ **False** 2. $(-8,-4)$: Test Point $\mathbf{x}=-6$; $\frac{3*\boxed{-6}}{16-\boxed{-6}^2}=\frac{-18}{-20}=\frac{9}{10}\overset{?}{\ge}\frac{1}{2}$ **True** 3. $(-4,2)$: Test Point $\mathbf{x}=0$; $\frac{3*\boxed{0}}{16-\boxed{0}^2}=0\overset{?}{\ge}\frac{1}{2}$ **False** 4. $(2,4)$: Test Point $\mathbf{x}=3$; $\frac{3*\boxed{3}}{16-\boxed{3}^2}=\frac{9}{7}0\overset{?}{\ge}\frac{1}{2}$ **True** 5. $(4,+\infty)$: Test Point $\mathbf{x}=10$; $\frac{3*\boxed{10}}{16-\boxed{10}^2}=\frac{30}{-84}=-\frac{5}{14}\overset{?}{\ge}\frac{1}{2}$ **False**	
7	Solution: $[-8,-4)\cup[2,4)$	

Graph of the solution set: