Inequalities
Radical

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We are looking for the numbers on the horizontal number line (x-axis) that make the inequalities containing radicals $\left(\sqrt{\boxed{?}}\ ;\ \sqrt[3]{\boxed{?}}\ ;\ \ldots\right)$ true. That is, when we are solving radical equations, we *only* consider solutions that are real numbers. We appeal to the following facts:

1. $\mathbf{u} < 0 \Rightarrow \sqrt{\mathbf{u}}$ is complex $\qquad$ $\mathbf{u} > 0 \Rightarrow \sqrt{\mathbf{u}}$ is real & positive

   0 $\qquad$ $\mathbf{u}$

   $$\mathbf{u} = 0 \Rightarrow \sqrt{\mathbf{u}} = 0$$

2. $\mathbf{u} < 0 \Rightarrow \sqrt[3]{\mathbf{u}}$ is real & negative $\qquad$ $\mathbf{u} > 0 \Rightarrow \sqrt[3]{\mathbf{u}}$ is real & positive

   0 $\qquad$ $\mathbf{u}$

   $$\mathbf{u} = 0 \Rightarrow \sqrt[3]{\mathbf{u}} = 0$$

3. Square Root : $\sqrt{\mathbf{u}} \geq 0$; $\sqrt{\mathbf{u}} \not< 0$

4. Cube Root: $\sqrt[3]{\mathbf{u}} \leq 0$ or $\sqrt[3]{\mathbf{u}} \geq 0$

Example 01: Solve for x: $\sqrt{3-4\mathbf{x}} \geq 0$
Solution:

Step	Inequality	Reason
0	$\sqrt{3-4\mathbf{x}} \geq 0$	$\Rightarrow 3\mathbf{x} - 4 \geq 0$
1	$3-4\mathbf{x} \geq 0$	
2	$3 \geq 4\mathbf{x}$	

3	$\frac{3}{4} \geq \mathbf{x} \quad \left(\text{or } x \leq \frac{3}{4}\right)$	

Solution in interval notation: $\left(-\infty, \frac{3}{4}\right]$

Graph of solution set:

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Example 02: Solve for x: $\sqrt{3 - 4\mathbf{x}} \leq 0$
Solution:

Step	Inequality	Reason
0	$\sqrt{3 - 4\mathbf{x}} \leq 0$	$\Rightarrow 3 - 4\mathbf{x} = 0$
1	$3 = 4\mathbf{x}$	
2	$\frac{3}{4} = \mathbf{x} \left(\text{or } \mathbf{x} = \frac{3}{4}\right)$	
3	Solution: $\left\{\frac{3}{4}\right\}$	

Graph of solution set:

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Example 03: Solve for x: $\sqrt[3]{3 - 4\mathbf{x}} < 0$
Solution:

Step	Inequality	Reason
0	$\sqrt[3]{3 - 4\mathbf{x}} < 0$	$\Rightarrow 3 - 4\mathbf{x} < 0$
1	$3 - 4\mathbf{x} < 0$	
2	$3 < 4\mathbf{x}$	
3	$\frac{3}{4} < \mathbf{x} \left(\text{or } \mathbf{x} > \frac{3}{4}\right)$	

Solution in interval notation: $\left(\frac{3}{4}, +\infty\right)$

Graph of solution set:

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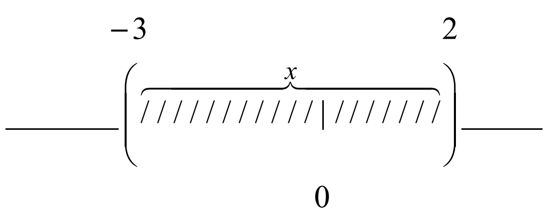
Example 04: Solve for x: $\sqrt{6-\mathbf{x}-\mathbf{x}^2} > 0$
Solution:

Step	Inequality	Reason
0	$\sqrt{6-\mathbf{x}-\mathbf{x}^2} > 0$	$\Rightarrow 6-\mathbf{x}-\mathbf{x}^2 > 0$
1	$6-\mathbf{x}-\mathbf{x}^2 > 0$	
2	$(3+\mathbf{x})(2-\mathbf{x}) > 0$	
3	Determine Boundary Points: $(3+\mathbf{x})(2-\mathbf{x}) = 0$ $\mathbf{x} = -3$; $\mathbf{x} = 2$	
4	Check Boundary Points: 1. $\mathbf{x} = -3 : (3+[-3])(2-[-3]) \overset{?}{>} 0$ $(0)(5) \overset{?}{>} 0$ $0 > 0$ False ; -3 is NOT in the solution set 2. $\mathbf{x} = 2 : (3+[2])(2-[2]) \overset{?}{>} 0$ $(5)(0) \overset{?}{>} 0$ $0 > 0$ False ; 2 is NOT in the solution set	

5	Check Intervals: 1. $(-\infty,-3)$: Test Point $\mathbf{x}=-6$; $(3+[-6])(2-[-6])\overset{?}{>}0$ $(-3)(8)\overset{?}{>}0$ $-24>0$ False ; $(-\infty,-3)$ is NOT in the solution set 2. $(-3,2)$: Test Point $\mathbf{x}=0$; $(3+[0])(2-[0])\overset{?}{>}0$ $(3)(2)\overset{?}{>}0$ $6>0$ True ; $(-3,2)$ is in the solution set 3. $(2,+\infty)$: Test Point $\mathbf{x}=10$; $(3+[10])(2-[10])\overset{?}{>}0$ $(13)(-8)\overset{?}{>}0$ $-104>0$ False ; $(2,+\infty)$ is NOT in the solution set	
6	Solution Set: $(-3,2)$	

Graph of the solution set:



Example 05: Solve for x: $\sqrt[3]{\dfrac{\mathbf{x}+4}{\mathbf{x}}}\le 4$

Solution:

Step	Inequality	Reason
0	$\sqrt[3]{\dfrac{\mathbf{x}+4}{\mathbf{x}}}\le 4$	
1	$\left(\sqrt[3]{\dfrac{\mathbf{x}+4}{\mathbf{x}}}\right)^3\le 4^3$	
2	$\dfrac{\mathbf{x}+4}{\mathbf{x}}\le 64$	
3	$\dfrac{\mathbf{x}+4}{\mathbf{x}}-\dfrac{64}{1}\le 0$	

4	$\dfrac{\mathbf{x}+4}{\mathbf{x}}-\dfrac{64\mathbf{x}}{\mathbf{x}}\le 0$	
5	$\dfrac{4-63\mathbf{x}}{\mathbf{x}}\le 0$	Standard Form
6	Boundary Points: 1) $4-63\mathbf{x}=0\Rightarrow \mathbf{x}=\dfrac{4}{63}$ in the solution set 2) $\mathbf{x}=0$ NEVER in the solution set	
7	Check Intervals: 1. $(-\infty,0)$: Test Point $\mathbf{x}=-10$; $\sqrt[3]{\dfrac{\boxed{-10}+4}{-10}}\overset{?}{\le}4$ $\sqrt[3]{\dfrac{-6}{-10}}=\sqrt[3]{\dfrac{3}{5}}\overset{?}{\le}4$ **True** 2. $\left(0,\dfrac{4}{63}\right)$: Test Point $\mathbf{x}=\dfrac{1}{63}$; $\sqrt[3]{\dfrac{\boxed{1/63}+4}{1/63}}\overset{?}{\le}4$ $\sqrt[3]{253}\overset{?}{\le}4$ **False** 3. $\left(\dfrac{4}{63},+\infty\right)$: Test Point $\mathbf{x}=10$; $\sqrt[3]{\dfrac{10+4}{10}}\overset{?}{\le}4$ $\sqrt[3]{\dfrac{10+4}{10}}=\sqrt[3]{\dfrac{7}{5}}\overset{?}{\le}4$ **True**	
8	Solution: $(-\infty,0)\cup\left[\dfrac{4}{63},+\infty\right)$	

Graph of the solution set: