Inequalities
Absolute Value

MATH by Wilson
Your Personal Mathematics Trainer
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There are two (2) types to be considered:

1. **Type 1**: $|\mathbf{u}| < \mathbf{b}$; $|\mathbf{u}| \le \mathbf{b}$ where $\mathbf{b} > 0$

   The solution of $|\mathbf{u}| < \mathbf{b}$; $\mathbf{b} > 0$ is $-\mathbf{b} < \mathbf{u} < \mathbf{b}$ - open interval

   The solution of $|\mathbf{u}| \le \mathbf{b}$; $\mathbf{b} > 0$ is $-\mathbf{b} \le \mathbf{u} \le \mathbf{b}$ - closed interval

Note: Solutions are open or closed intervals

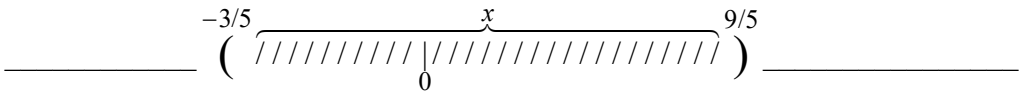
Example 01: Solve for x: $|5\mathbf{x} - 3| < 6$

Solution:

Step	Equation	Reason
0	$\lvert 5\mathbf{x} - 3\rvert < 6$	$\mathbf{u} = 5\mathbf{x} - 3$
1	$-6 < 5\mathbf{x} - 3 < 6$	
2	$-3 < 5\mathbf{x} < 9$	
3	$-\frac{3}{5} < \mathbf{x} < \frac{9}{5}$	

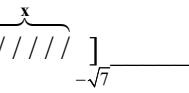
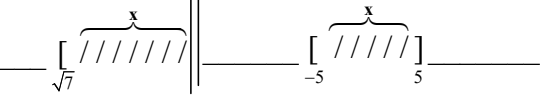
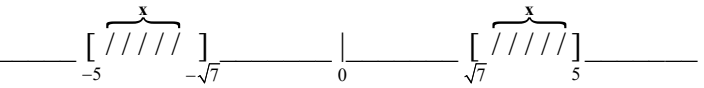
Solution in interval notation: $\left(-\frac{3}{5}, \frac{9}{5}\right)$

Graph of the solution set:



Equation 02: Solve for x: $\left|\mathbf{x}^2-16\right|\le 9$

Solution:

Step	Equation	Reason
0	$\left\|\mathbf{x}^2-16\right\|\le 9$	$\mathbf{u}=\mathbf{x}^2-16$
1	$-9\le \mathbf{x}^2-16\le 9$	
2	$-9\le \mathbf{x}^2-16$ $0\le \mathbf{x}^2-7$ $0\le\left(\mathbf{x}-\sqrt{7}\right)\left(\mathbf{x}+\sqrt{7}\right)$  $\mathbf{x}^2-16\le 9$ $\mathbf{x}^2-25\le 0$ $(\mathbf{x}-5)(\mathbf{x}+5)\le 0$ 	**Trade For Two Quadratic Inequalities**
3	Solutions MUST satisfy both inequalities: 	
4	**Solution:** $\left[-\sqrt{7},-5\right]\cup\left[\sqrt{7},5\right]$	

2. **Type 2**: $\left|\mathbf{u}\right|>\mathbf{b}$; $\left|\mathbf{u}\right|\ge\mathbf{b}$ where $\mathbf{b}>0$

The solution of $\left|\mathbf{u}\right|>\mathbf{b}$; $\mathbf{b}>0$ satisfies $\mathbf{u}<-\mathbf{b}$ **OR** $\mathbf{u}>\mathbf{b}$:

$$(-\infty,-\mathbf{b})\cup(\mathbf{b},+\infty)$$

The solution of $\left|\mathbf{u}\right|\ge\mathbf{b}$; $\mathbf{b}>0$ satisfies $\mathbf{u}\le-\mathbf{b}$ **OR** $\mathbf{u}\ge\mathbf{b}$

$$(-\infty,-\mathbf{b}]\cup[\mathbf{b},+\infty)$$

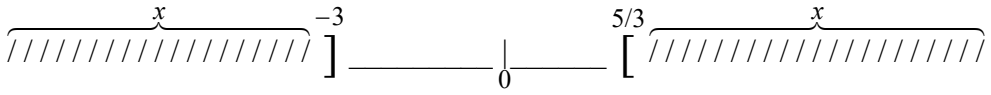
Note: Solutions are unions of open or closed intervals

Examples 03: Solve for x: $|2+3\mathbf{x}| \geq 7$

Solution:

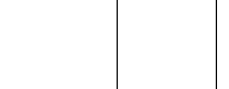
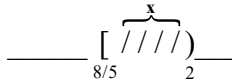
Step	Equation	Reason
0	$\lvert 2+3\mathbf{x}\rvert \geq 7$	
1	$2+3\mathbf{x} \leq -7$ $3\mathbf{x} \leq -9$ $\mathbf{x} \leq -3$ $\Vert$ $2+3\mathbf{x} \geq 7$ $3\mathbf{x} \geq 5$ $\mathbf{x} \geq \frac{5}{3}$	**Trade For Two Linear Inequalities**
2	Solution: $(-\infty,-3]\cup\left[\frac{5}{3},+\infty\right)$	

Graph of the solution set:



Equation 04: Solve for x: $\left|\frac{\mathbf{x}}{\mathbf{x}-2}\right| \geq 4$

Solution:

Step	Equation	Reason
0	$\left\lvert\frac{\mathbf{x}}{\mathbf{x}-2}\right\rvert \geq 4$	
1	$\frac{\mathbf{x}}{\mathbf{x}-2} \leq -4$ $\frac{\mathbf{x}}{\mathbf{x}-2}+\frac{4(\mathbf{x}-2)}{\mathbf{x}-2} \leq 0$ $\frac{\mathbf{x}+4\mathbf{x}-8}{\mathbf{x}-2} \leq 0$ $\frac{5\mathbf{x}-8}{\mathbf{x}-2} \leq 0$  $\Vert$ $\frac{\mathbf{x}}{\mathbf{x}-2} \geq 4$ $\frac{\mathbf{x}}{\mathbf{x}-2}-\frac{4(\mathbf{x}-2)}{\mathbf{x}-2} \geq 0$ $\frac{\mathbf{x}-4\mathbf{x}+8}{\mathbf{x}-2} \geq 0$ $\frac{8-3\mathbf{x}}{\mathbf{x}-2} \geq 0$ 	**Trade For Two Rational Inequalities**
2	Solution: $\left[\frac{8}{5},2\right)\cup\left(2,\frac{8}{3}\right]$	