FUNctions – Introduction (Formal)

$$y = \overset{\text{Name}}{\underbrace{\mathbf{f}\,(\underset{\text{Input}}{\mathbf{x}})}_{\text{Output}}} = \text{formula}$$

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Informal:

Basic FUNction Idea/Concept – Allowable Input $\overset{\text{Implies}}{\Rightarrow}$ Unique Output

Formal:
Definition: A **function** $\mathbf{f}$ from a set $\mathbf{X} \subseteq \mathbb{R}_x$ unto a set $\mathbf{Y} \subseteq \mathbb{R}_y$ is a correspondence that associates with each $\mathbf{x} \in \mathbf{X}$ one and only one $\mathbf{y} \in \mathbf{Y}$

Note: The symbol "$\subseteq$" means "contained in" or "is a subset of".
The symbol "$\in$" means "is an element or a member of".

$\mathbf{X} \subseteq \mathbb{R}_x$ is called the **domain**: **Dom f**

Note: The domain of $\mathbf{f}$ is a subset of all the real numbers on the *horizontal* number line: x-axis

As we will see in more detail soon, the **domain** of $\mathbf{f}$ may be defined two (2) ways:

2. **Implicitly** – the domain is defined by $\mathbf{Dom\ f} = \{\mathbf{x} \in \mathbb{R}_{\mathbf{x}} \overset{\text{Such That}}{|} \mathbf{f}(\mathbf{x}) \in \mathbb{R}_{\mathbf{y}}\}$

1. **Explicitly** – the domain is given

$\mathbf{Y} \subseteq \mathbb{R}_{\mathbf{y}}$ is called the **range**: **Range f**

Note: The range of **f** is a subset of all the real numbers on the *vertical* number line: y-axis

Assumption: We consider **real-valued functions**, that is, functions that take real #'s to real #'s

What do we *officially* need to specify a function?

1. Nameor Type: **Identity, Square, Absolute Value, Exponential, …**
2. Symbol: $\mathbf{f}, \mathbf{g}, \mathbf{h}, \ldots$
3. Domain: **Dom f**
4. Correspondence (rule, formula, table, …): $\mathbf{f}(\mathbf{x}), \mathbf{g}(\mathbf{x}), \mathbf{h}(\mathbf{x}), \ldots$
5. Range: **Range f**

We usually are NOT given all these five (5) items when a function is defined. We have certain rules to follow to find out what is not given.

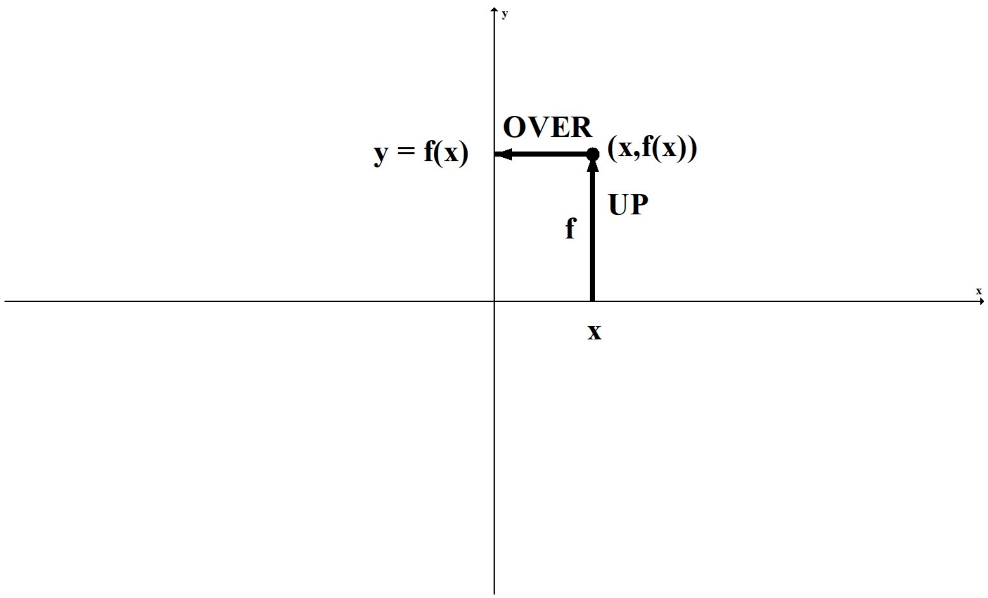
We frequently construct two (2) dimensional representations of our functions with what is called their **graph**:

Definition: The **<u>graph</u>** of a function **f** is a set of ordered pairs:

$$\{(\mathbf{x}, \mathbf{f}(\mathbf{x})) \mid \mathbf{x} \in \mathbf{Dom\ f}\}$$

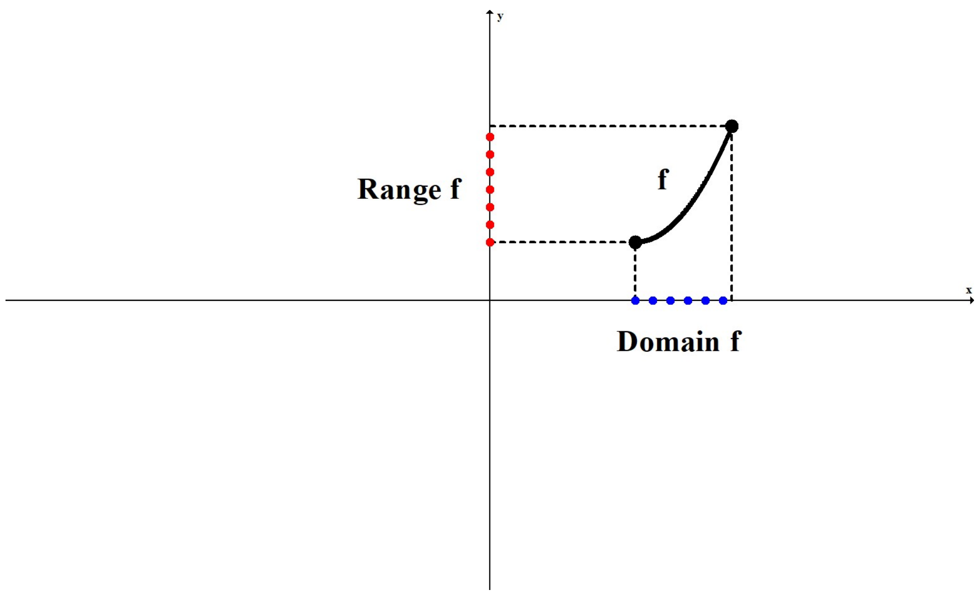
It is constructed (drawn) in the x-y Number Plane (Rectangular Coordinate System, Cartesian Coordinate System).

Note: The correspondence is seen by starting with an x in the domain on the x-axis, going up (or down) to the graph, and then projecting this point onto the y-axis.



Note: The domain can be *any* subset on the x-axis and the range can be *any* subset on the y-axis. If we "plot" *all* the ordered pairs (x , f(x)) we say that we have "graphed" the function **f**.

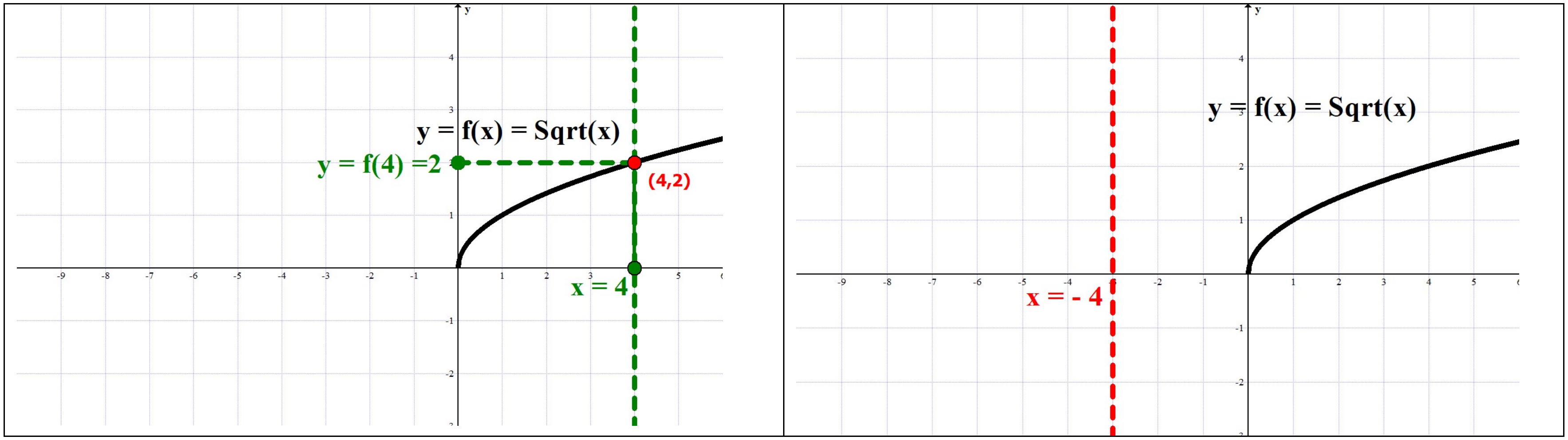
Let's look at the graph of **f** from a geometrical point of view. As the figure below indicates, the **domain** is the projection of the graph onto the x-axis and the **range** is the projection onto the y-axis:



If we draw a *vertical line* through an x-value on the horizontal number line and it intersects the graph, then that x-value **IS** in the **Dom f**. Otherwise, it's NOT in the **Dom f**:

So, if we are given an x-value in **Dom f**, to find its corresponding y-value, we just follow the vertical line to wherever it intersects the graph and project this point over to the y-axis and we'll have the corresponding y-value.

Below we see that $x = 4$ corresponds to $y = 2$ in the left hand graph and $x = -4$ is NOT in the domain in the right hand graph..



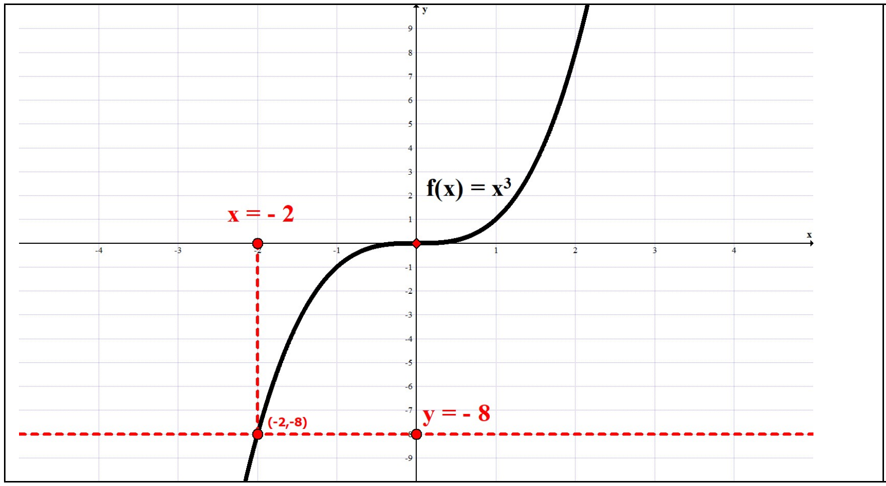
If we draw a *horizontal line* through an "y" value on the vertical number line and it intersects the graph, then that "y" value **IS** in the **Rng f**. Otherwise, it's NOT in the **Rng f**:

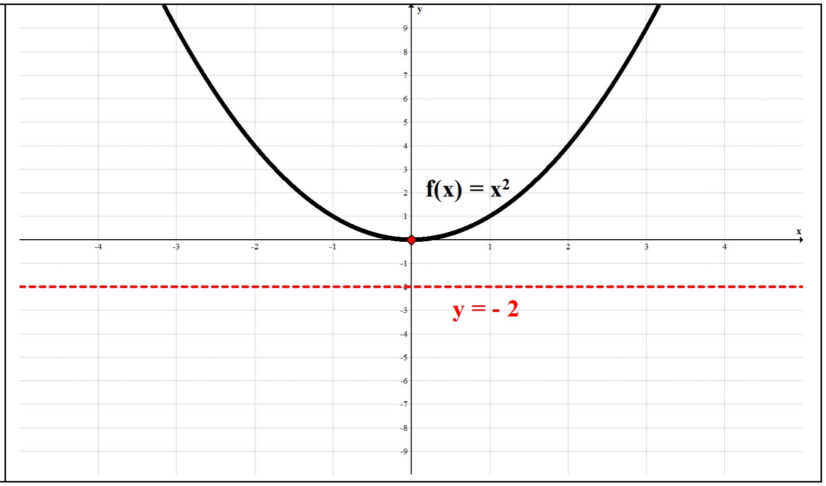
So, if we are given an y value in **Rng f**, to find its corresponding x value, we just follow the horizontal line to wherever it intersects the graph and project this point over to the x-axis and we'll have the corresponding x value.

In the figure below

a. The graph on the *left* shows that y = -8 **IS** in the range of $\mathbf{f}(\mathbf{x}) = \mathbf{x}^3$ since $-8 = \mathbf{y} = \mathbf{f}(-2)$.

b. The graph on the *right* shows that y = - 2 **IS NOT** in the range of $\mathbf{f}(\mathbf{x}) = \mathbf{x}^2$.


f(x) = x³
x = - 2
(-2,-8)
y = - 8


f(x) = x²
y = - 2

