

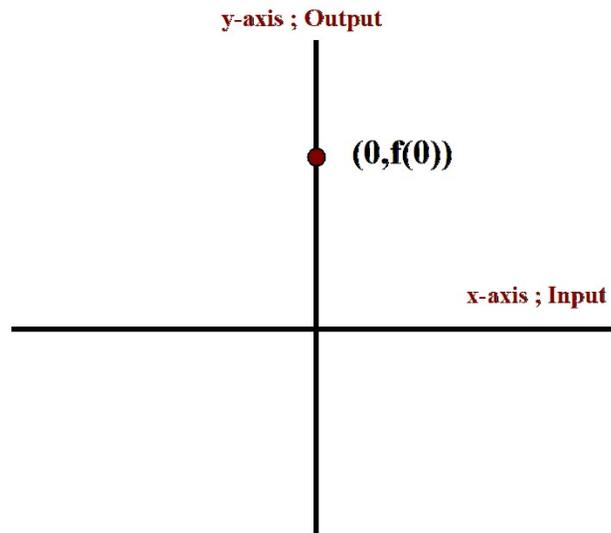
FUNctions: Intercept Points

[Points of the form $(0,?)$ & $(?,0)$]

[Intersection of graph with x-axis & y-axis]

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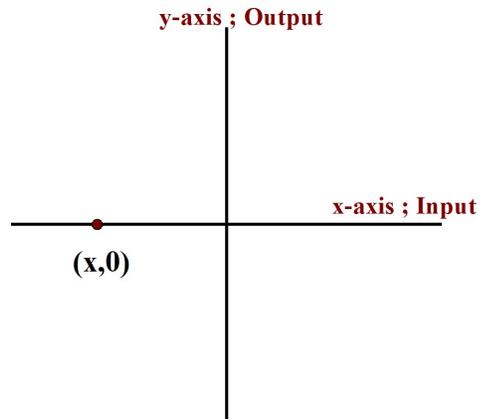
Definition: If $0 \in \text{Dom } f$, then *the* point $(0, f(0))$ is *the* **y-intercept point** of the function **f**:



Key Facts:

1. There is a maximum of one (1) y-intercept point
2. It is the intersection of the graph with the y-axis: $x = 0$
3. The Action Verb is **EVALUATE** – Calculate $f(0)$: $(0, f(0))$

Definition: If $f(x) = 0$ for some x in the domain of a function f , then the point $(x,0)$ is *an x-intercept point* of the function f :



Key Facts:

1. There are $0,1,2,\dots,n,\dots$ up to an infinite number of x-intercept points:

$$\{x \in \text{Dom } f \mid y = f(x) = 0\}$$

2. It is an intersection of the graph with the x-axis
3. The Action Verb is **Solve**: Solve the equation $f(x) = 0$ for x

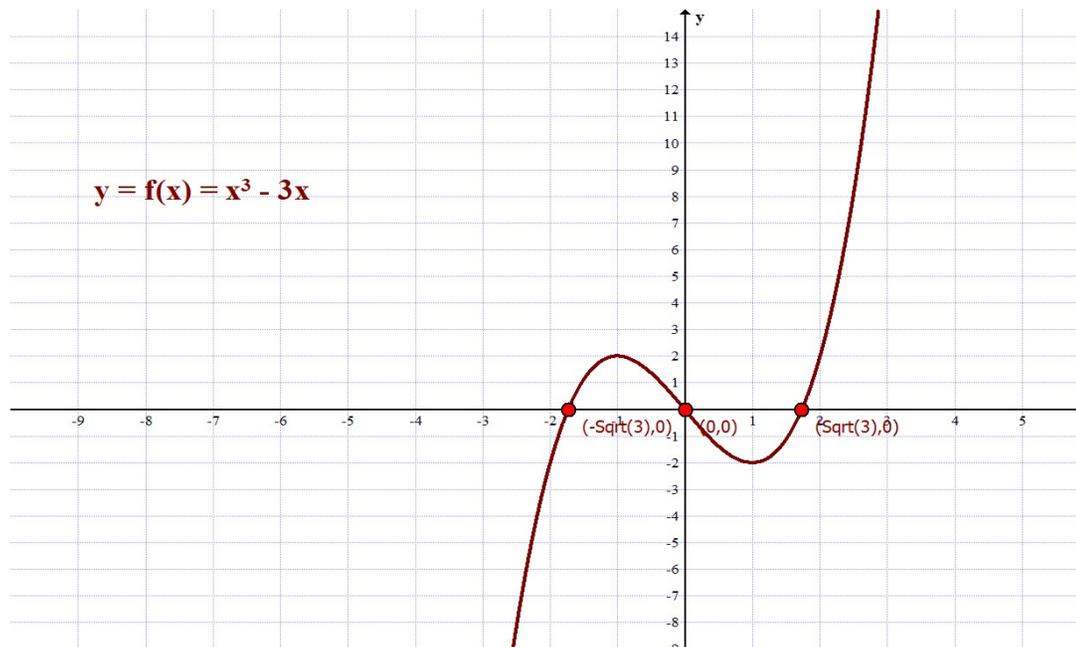
In the example below, we draw the entire graph but at this point of our analysis we can only find the intercept point(s).

Example : Find the x-intercept and y-intercept points of the following functions:

1. $f(x) = x^3 - 3x$

The y-intercept point is $(0,0)$. The x-intercept points are given by

$$f(x) = x(x^2 - 3) \stackrel{\text{SET}}{=} 0 \Rightarrow x = \pm\sqrt{3}, 0 \Rightarrow (-\sqrt{3}, 0), (0, 0), (\sqrt{3}, 0)$$

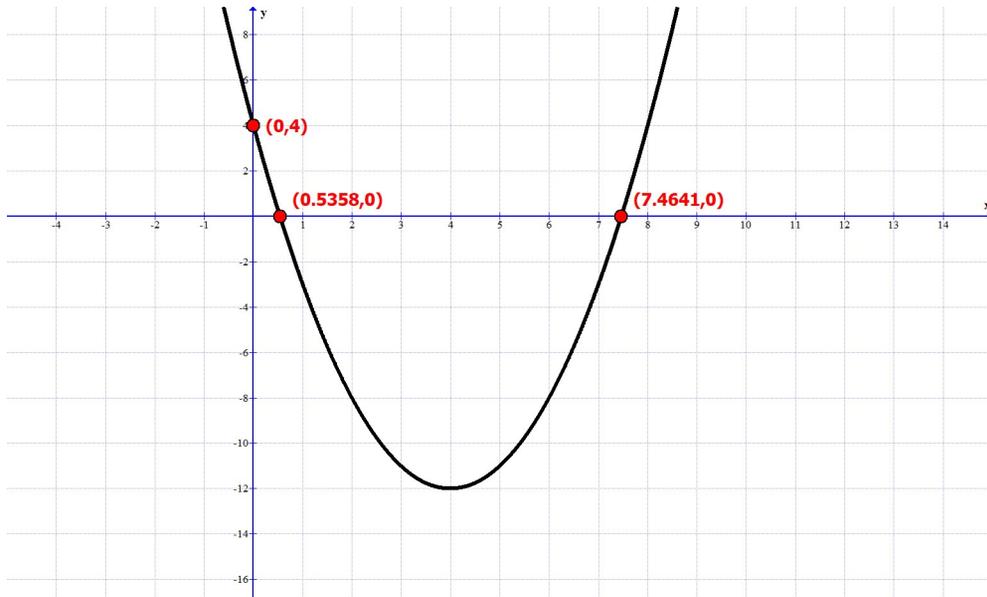


Note: (0,0) is both the x-intercept point and the y-intercept point.

2. $f(x) = 4 - 8x + x^2$

The y-intercept point is (0,4). The x-intercept points are given by

$$f(x) = 4 - 8x + x^2 \stackrel{\text{SET}}{=} 0 \Rightarrow x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(4)}}{2} = \frac{8 \pm \sqrt{48}}{2} = \frac{8 \pm 4\sqrt{3}}{2}$$
$$\Rightarrow x = 4 \pm 2\sqrt{3} \Rightarrow (4 - 2\sqrt{3}, 0), (4 + 2\sqrt{3}, 0)$$

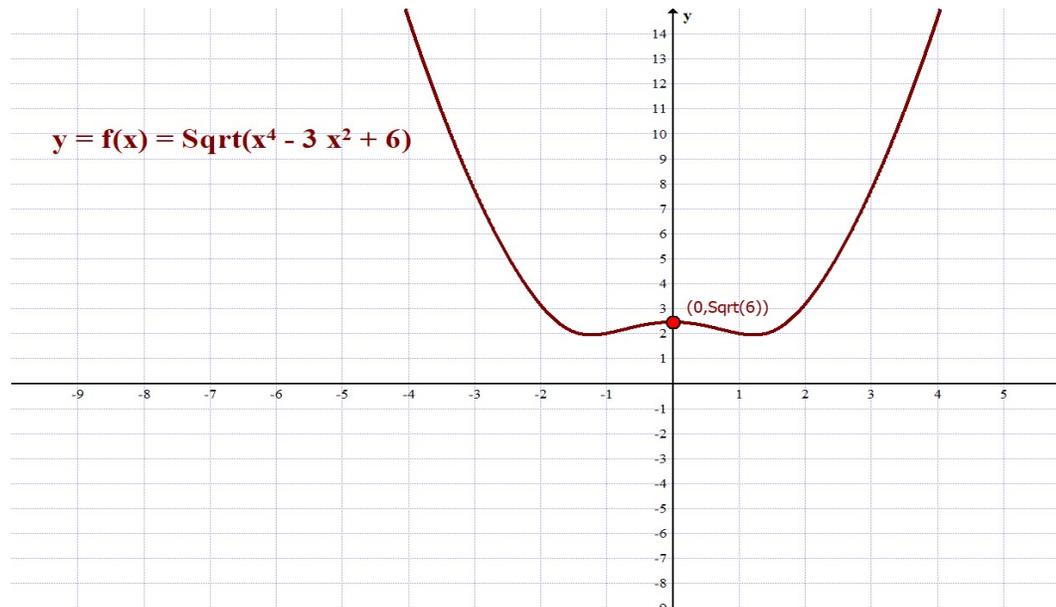


3. $f(x) = \sqrt{x^4 - 3x^2 + 6}$

The y-intercept point is $(0, \sqrt{6})$. The x-intercept points are given by

$$f(x) = \sqrt{x^4 - 3x^2 + 6} \stackrel{\text{SET}}{=} 0 \Rightarrow x^4 - 3x^2 + 6 = 0 \Rightarrow x^2 = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(6)}}{2}$$

$$\Rightarrow x^2 = \frac{3 \pm \sqrt{-15}}{2} \in \mathbb{C} \text{ (Complex Numbers)} \Rightarrow \text{No x-intercept points}$$



4. $f(x) = \frac{2x}{x^2 + 4}$

The y-intercept point is (0,0). The x-intercept point(s) are given by

$$f(x) = \frac{2x}{x^2 + 4} = 0 \stackrel{\text{SET}}{\Rightarrow} x = 0 \Rightarrow (0,0)$$

