

FUNctions Odd/Even

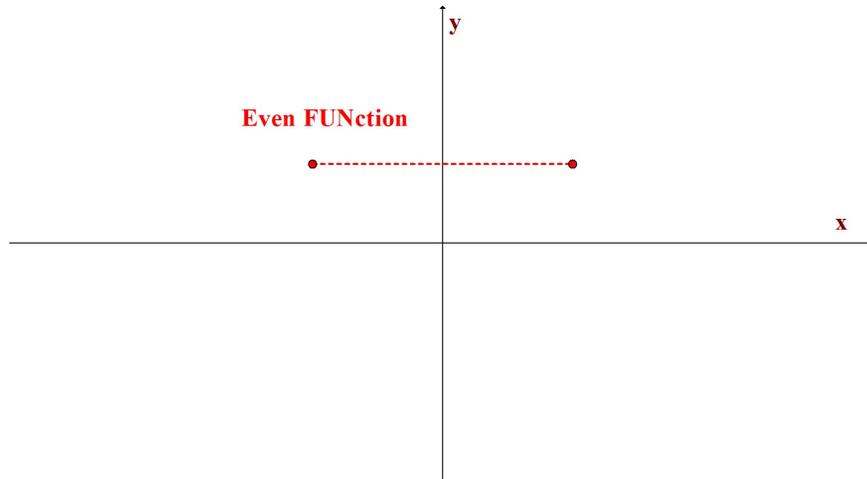
[Odd: $f(-x) = -f(x)$: graph symmetric wrt $(0,0)$]

[Even: $f(-x) = f(x)$: graph symmetric wrt $x = 0$]

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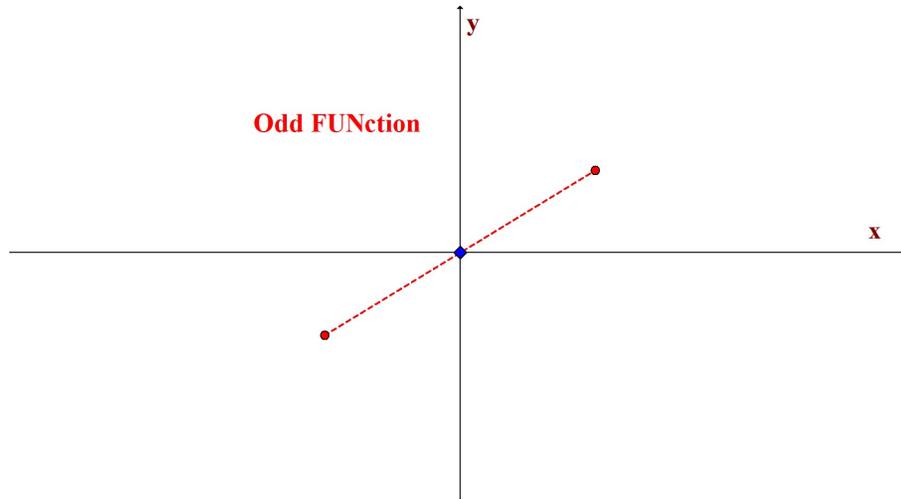
Definition: A function f is **even** if $f(-x) = \overset{\text{Manipulate}}{\dots} = f(x)$ for all $x \in \mathbf{Dom f}$.

Note: An *even* function will have a graph that is symmetric with respect to the y-axis.



Definition: A function f is **odd** if $f(-x) = \overset{\text{Manipulate}}{\dots} = -f(x)$ for all $x \in \text{Dom } f$.

Note: An *odd* function will have a graph that is symmetric with respect to the Origin: (0,0)



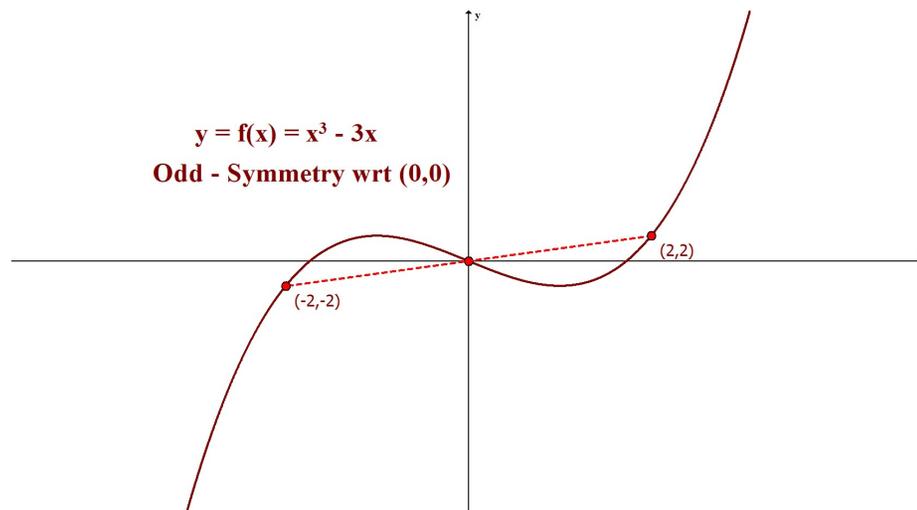
We will test the following functions to determine if they are odd, even or neither and then show their graphs to visually demonstrate the appropriate symmetry.

Example: Check the following functions to see if they are even, odd, or neither.

a. $f(x) = x^3 - 3x$

We have $f(-x) = (-x)^3 - 3(-x) = -x^3 + 3x = -(x^3 - 3x) = -f(x) \Rightarrow$ **ODD**

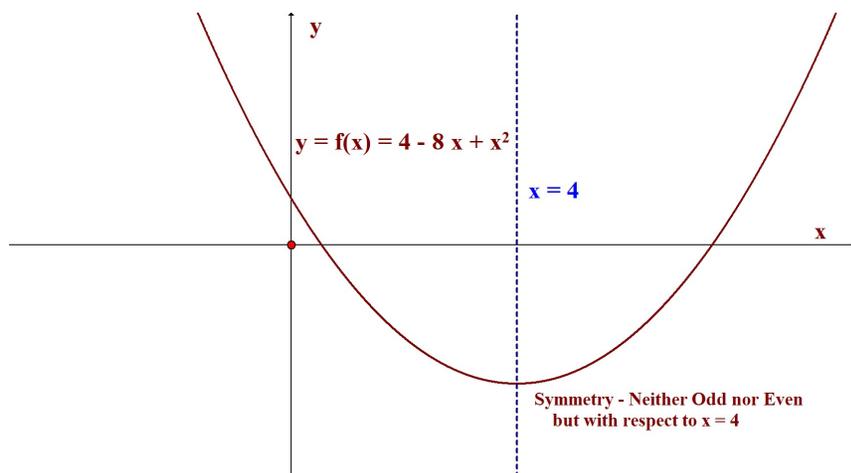
We know the graph is symmetric with respect to the Origin (0,0) as its graph below shows:



b. $f(x) = 4 - 8x + x^2$

We have $f(-x) = 4 - 8(-x) + (-x)^2 = 4 + 8x + x^2 \neq \begin{cases} f(x) \\ -f(x) \end{cases} \Rightarrow \text{Neither}$

We know the graph is NOT symmetric with respect to the Origin (0,0) or the y-axis ($x = 0$) as shown below:

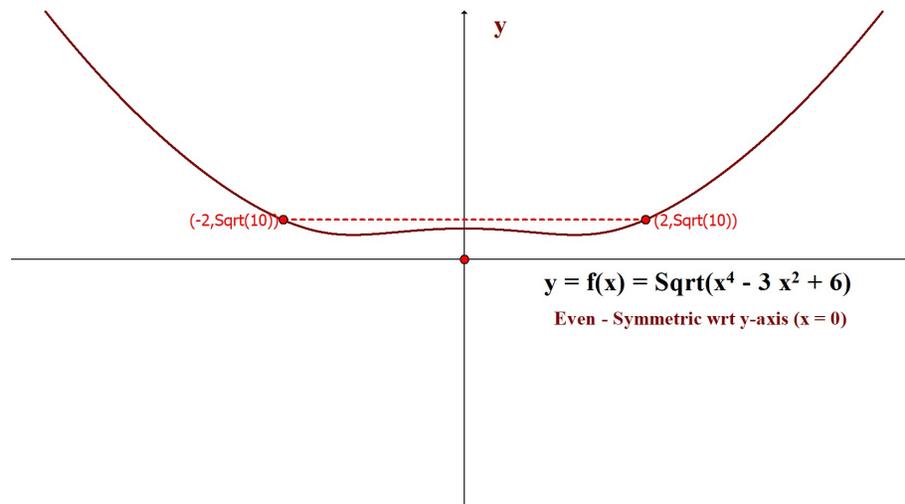


Note: This quadratic function is symmetric wrt the vertical line $x = 4$.

c. $f(x) = \sqrt{x^4 - 3x^2 + 6}$

We have $f(-x) = \sqrt{(-x)^4 - 3(-x)^2 + 6} = \sqrt{x^4 - 3x^2 + 6} = f(x) \Rightarrow$ **EVEN**

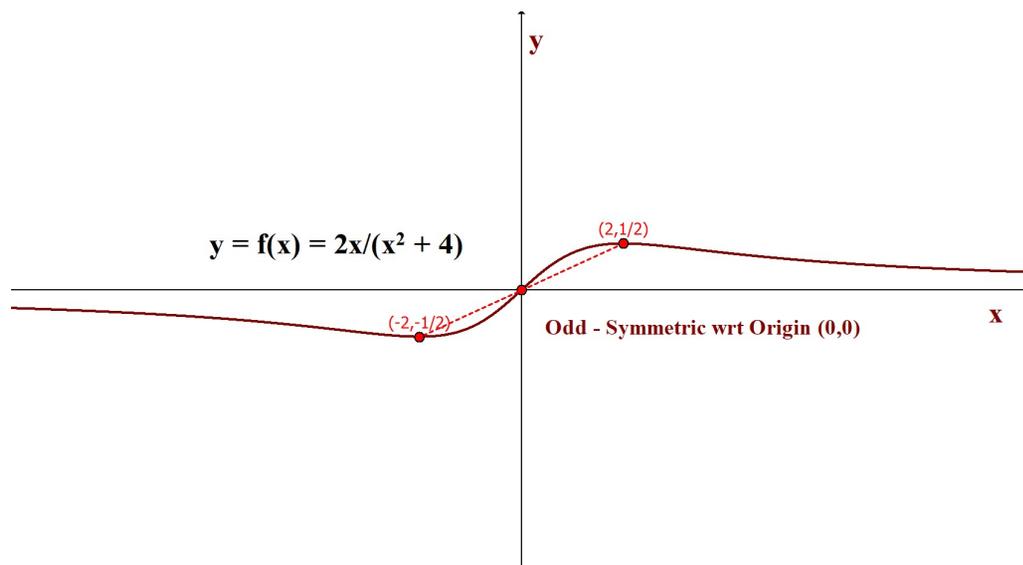
We know the graph is symmetric with respect to the y-axis ($x = 0$) as shown below:



d. $f(x) = \frac{2x}{x^2 + 4}$

We have $f(-x) = \frac{2(-x)}{(-x)^2 + 4} = \frac{-2x}{x^2 + 4} = -\frac{2x}{x^2 + 4} = -f(x) \Rightarrow$ **ODD**

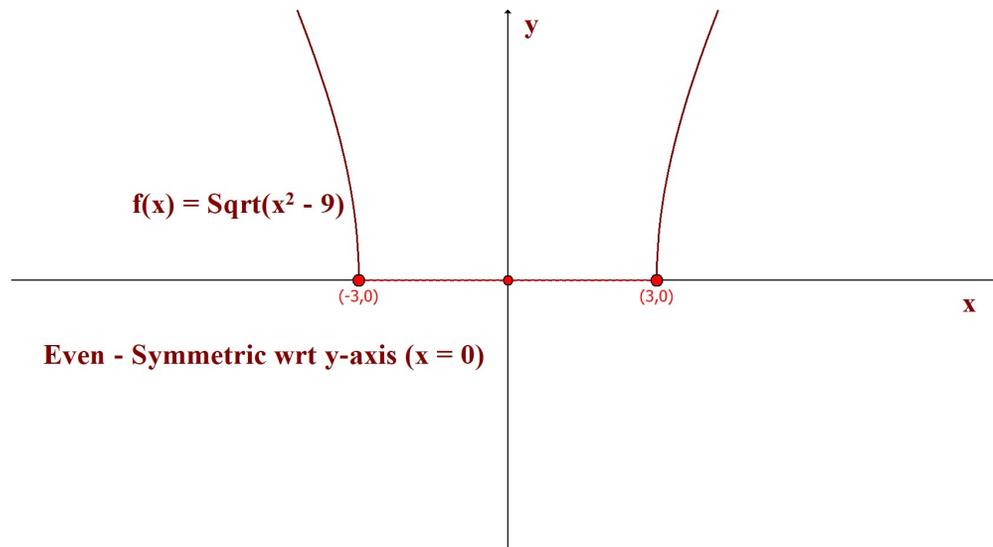
We know the graph is symmetric with respect to the Origin (0,0) as shown below:



e. $f(x) = \sqrt{x^2 - 9}$

We have $f(-x) = \sqrt{(-x)^2 - 9} = \sqrt{x^2 - 9} = f(x) \Rightarrow$ **EVEN**

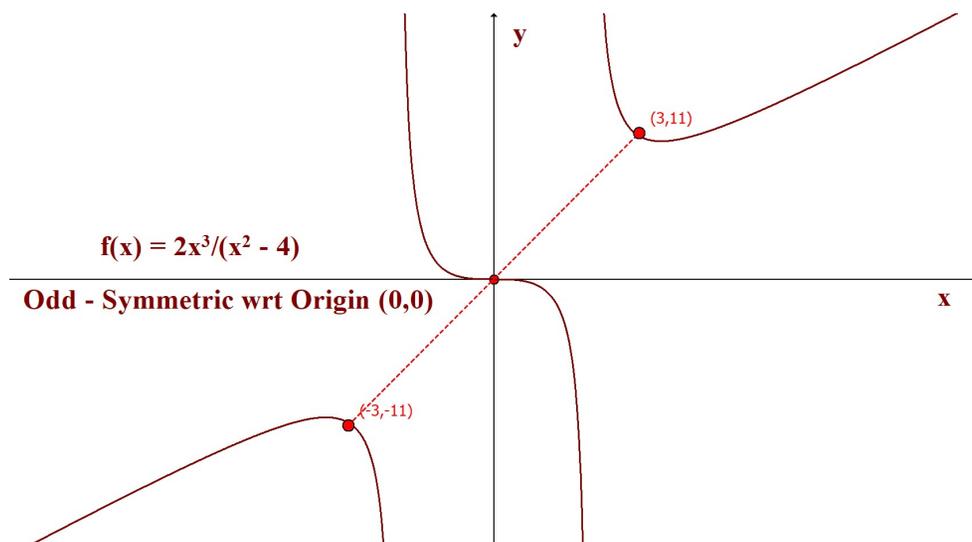
We know the graph is symmetric with respect to the y-axis as shown below:



f. $f(x) = \frac{2x^3}{x^2 - 4}$

We have $f(-x) = \frac{2(-x)^3}{(-x)^2 - 4} = -\frac{2x^3}{x^2 - 4} = -f(x) \Rightarrow$ **ODD**

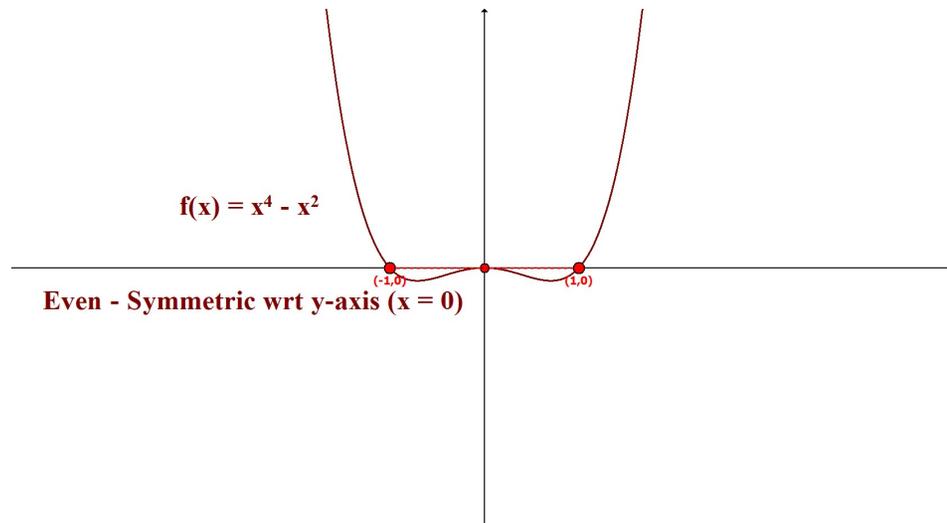
We know the graph is symmetric with respect to Origin (0,0) as shown below:



g. $f(x) = x^4 - 2x^2$

We have $f(-x) = (-x)^4 - (-x)^2 = x^4 - x^2 = f(x) \Rightarrow$ **EVEN**

We know the graph is symmetric with respect to the y-axis as shown below:



h. $f(x) = 3 - 5x^3 + 3x^5$

We have $f(-x) = 3 - 5(-x)^3 + 3(-x)^5 = 3 + 5x^3 - 3x^5 \neq \begin{cases} f(x) \\ -f(x) \end{cases} \Rightarrow \text{Neither}$

We know the graph is NOT even or odd as shown below:

