

Linear Functions - Equations of Lines

$$[y = f(x) = m x + b]$$

Slope y-intercept

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Your Personal Mathematics Trainer
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Linear Function Form (Slope & y-Intercept): $y = f(x) = mx + b$

Note: $x = 0 \Rightarrow y = f(0) = m \cdot 0 + b = b \Rightarrow (0, b)$ is the y-intercept point.

The letter **m** represents the “slope” of the “line” formed by $y = f(x) = mx + b$. For if we pick two different points

(x_1, y_1) & (x_2, y_2) on the graph of **f** and find $\frac{\Delta y}{\Delta x} = \frac{\text{Change in the y-values}}{\text{Change in the x-values}}$, we obtain

$$\begin{aligned} \frac{\text{Change in the y-values}}{\text{Change in the x-values}} &= \frac{f(x_2) - f(x_1)}{x_2 - x_1} \\ &= \frac{(mx_2 + b) - (mx_1 + b)}{x_2 - x_1} \\ &= \frac{m(x_2 - x_1)}{(x_2 - x_1)} \\ &= m \end{aligned}$$

Slope

So, for any two points, the $\frac{\text{Change in the y-values}}{\text{Change in the x-values}}$ is ALWAYS the same constant **m**. Therefore, its graph is a straight line with slope **m**. If **m** = 0, we obtain a horizontal line: **y = b**

Consider $y = f(x) = mx + b = -\frac{3}{2}x + 4$. We'll now find four (4) important properties that f possesses.

Properties:

1. Domain: **Dom** $f = \mathbb{R}_x$; there are no real numbers to reject.

2. Intercepts:

a. **y**: Set $x = 0$

$$f(0) = 4 \Rightarrow (0, 4) = (0, \mathbf{b}) ; \text{y-intercept point}$$

b. **x**: Set $y = f(x) = 0$

$$0 \stackrel{\text{SET}}{=} y = f(x) = -\frac{3}{2}x + 4$$

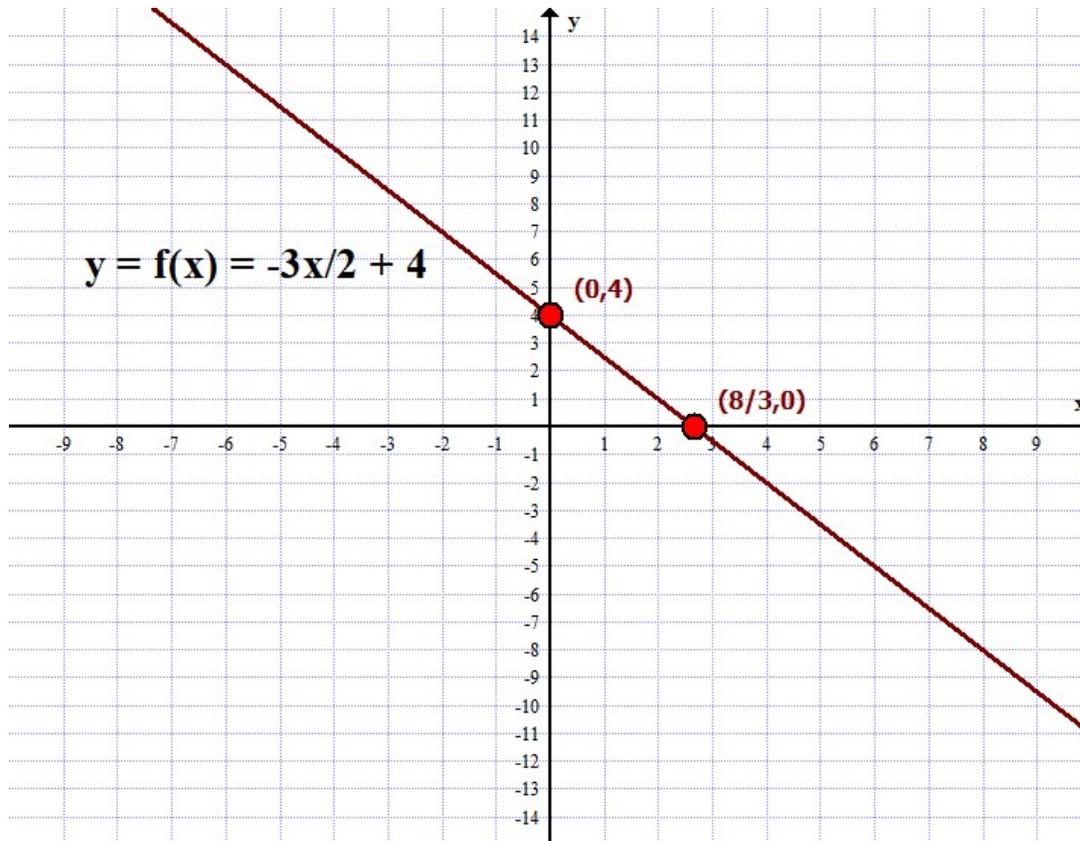
$$\frac{3}{2}x = 4$$

$$x = \frac{8}{3} \Rightarrow \left(\frac{8}{3}, 0\right) ; \text{x-intercept point}$$

3. Slope: $\mathbf{m} = -\frac{3}{2}$

4. Range: **Range** $f = \mathbb{R}_y$, note the **range** is the projection of the graph onto the y-axis.

Drawing a straight line through the two (2) intercept points, we obtain the graph of f :



Note: The slope is *negative* so the line is slanted downward.

Although a straight line may be represented by $y = mx + b$, there are several other ways to represent a line:

Given two (2) points $P(x_1, y_1)$ and $Q(x_2, y_2)$ so that the slope is $m = \frac{y_2 - y_1}{x_2 - x_1}$ or one point $P(x_1, y_1)$ and a slope m ,

there are four (4) forms the equation of a straight line using these data can take:

1. **Standard Form:** $Ax + By = C$

a. Two Points: $A = y_2 - y_1$; $B = x_1 - x_2$; $C = x_2y_1 - x_1y_2$

b. Point & Slope: $A = m$; $B = -1$; $C = mx_1 - y_1$

2. **Slope & y-Intercept Form:** $y = mx + b$; $m = \frac{y_2 - y_1}{x_2 - x_1}$; $b = y_1 - \frac{y_2 - y_1}{x_2 - x_1}x_1$ (Linear Function Form)

3. **Point & Slope Form:** $y - y_1 = m(x - x_1)$; $m = \frac{y_2 - y_1}{x_2 - x_1}$ if two points are given

4. **Two Point Form:** $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Example 01: Consider the straight line $4x + 7y = 24$. Put this line in “linear function form” and find the properties of the function f .

Solution:

We first solve $4x + 7y = 24$ for y :

Step	Equation	Comments
0	$4x + 7y = 24$	$y = ?$
1	$7y = 24 - 4x$	
2	$y = \frac{24 - 4x}{7} = -\frac{4}{7}x + \frac{24}{7}$	
3	$y = f(x) = -\frac{4}{7}x + \frac{24}{7}$ $= mx + b$	

Properties:

1. Domain: $\text{Dom } f = \mathbb{R}_x$

2. Intercepts:

a. y : Set $x = 0$

$$f(0) = \frac{24}{7} \Rightarrow \left(0, \frac{24}{7}\right) = (0, b); \text{ y-intercept point}$$

b. x : Set $y = f(x) = 0$

$$0 \stackrel{\text{SET}}{=} y = f(x) = -\frac{4}{7}x + \frac{24}{7}$$

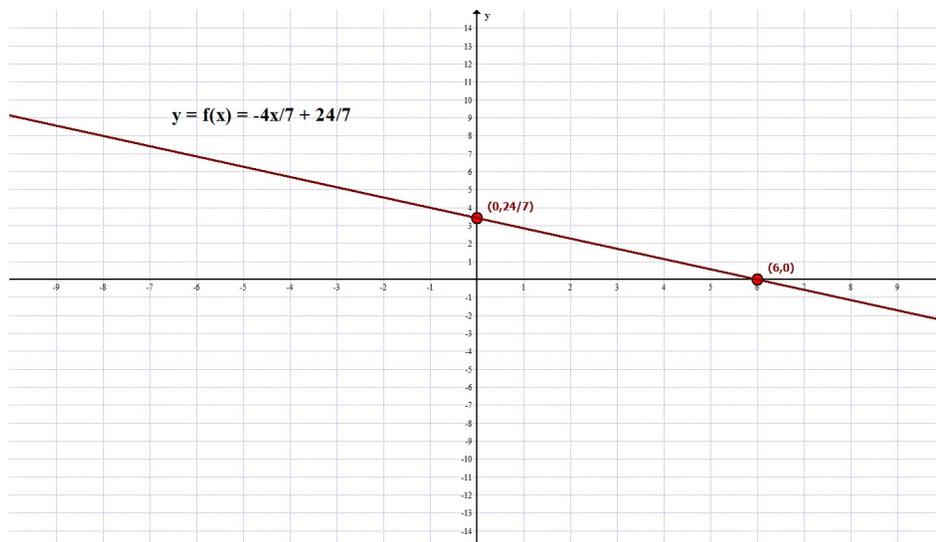
$$\frac{4}{7}x = \frac{24}{7}$$

$$x = \frac{24}{4} = 6 \Rightarrow (6,0) ; \text{x-intercept point}$$

3. Slope: $m = -\frac{4}{7}$

4. Range: $\text{Range } f = \mathbb{R}_y$

Below, we have the graph of f :



Example 02: Consider the straight line defined by $m = \frac{5}{3}$ and $P(-4, 7)$. Put this line in “linear function form” and find the properties of the function f .

Solution:

Using the Point & Slope Form, we obtain:

Step	Calculation	Comments
0	$y - y_1 = m(x - x_1)$	Point-slope form
1	$y - [7] = \left[\frac{5}{3}\right](x - [-4])$	
2	$y = \frac{5}{3}x + \frac{20}{3} + 7 = \frac{5}{3}x + \frac{41}{3}$	
3	$y = f(x) = \frac{5}{3}x + \frac{41}{3}$ $= \mathbf{mx + b}$	

Properties:

1. Domain: $\mathbf{Dom\ f} = \mathbb{R}_x$

2. Intercepts:

a. y : Set $x = 0$

$$f(0) = \frac{41}{3} \Rightarrow \left(0, \frac{41}{3}\right) = (0, \mathbf{b}); \text{ y-intercept point}$$

b. x : Set $y = f(x) = 0$

$$y = f(x) = \frac{5}{3}x + \frac{41}{3} = 0$$

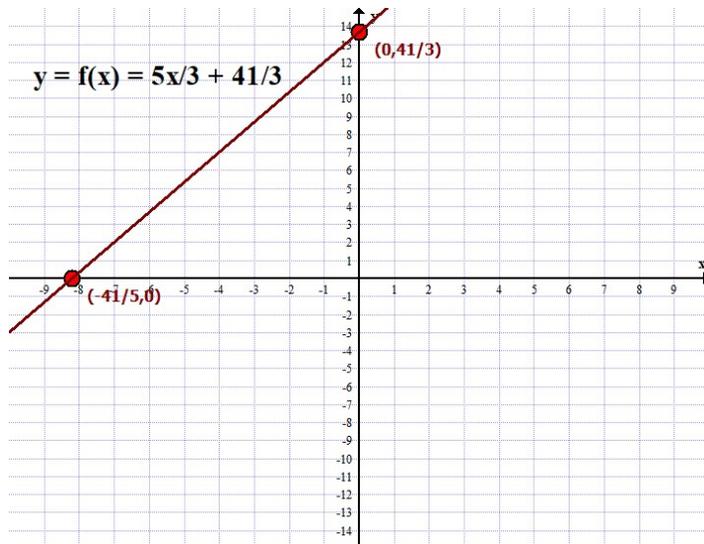
$$\frac{5}{3}x = -\frac{41}{3}$$

$$x = -\frac{41}{5} \Rightarrow \left(-\frac{41}{5}, 0\right); \text{ x-intercept point}$$

3. Slope: $m = \frac{5}{3}$; given

4. Range: **Range** $f = \mathbb{R}_y$

The graph is below:



Example 03: Consider the straight line defined by $P(-3,2)$ and $Q(5,9)$. Put this line in “linear function form” and find the properties of the function f .

Solution:

The slope is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{[9] - [2]}{[5] - [-3]} = \frac{7}{8}$. Now, using the Point & Slope Form, we have:

Step	Calculation	Comments
0	$y - y_1 = m(x - x_1)$	Point-slope form
1	$y - [2] = \left[\frac{7}{8}\right](x - [-3])$	
2	$y = \frac{7}{8}x + \frac{21}{8} + 2 = \frac{7}{8}x + \frac{37}{8}$	
3	$y = f(x) = \frac{7}{8}x + \frac{37}{8}$	

Properties:

1. Domain: $\text{Dom } f = \mathbb{R}_x$

2. Intercepts:

a. y : Set $x = 0$

$$f(0) = \frac{37}{8} \Rightarrow \left(0, \frac{37}{8}\right) = (0, \mathbf{b})$$

b. x : Set $y = f(x) = 0$

$$y = f(x) = \frac{7}{8}x + \frac{37}{8} = 0$$

$$\frac{7}{8}x = -\frac{37}{8}$$

$$x = -\frac{37}{7} \Rightarrow \left(-\frac{37}{7}, 0\right)$$

3. Slope: $m = \frac{7}{8}$

4. Range: **Range** $f = \mathbb{R}_y$

The graph is below:

