

Linear Functions - More Topics

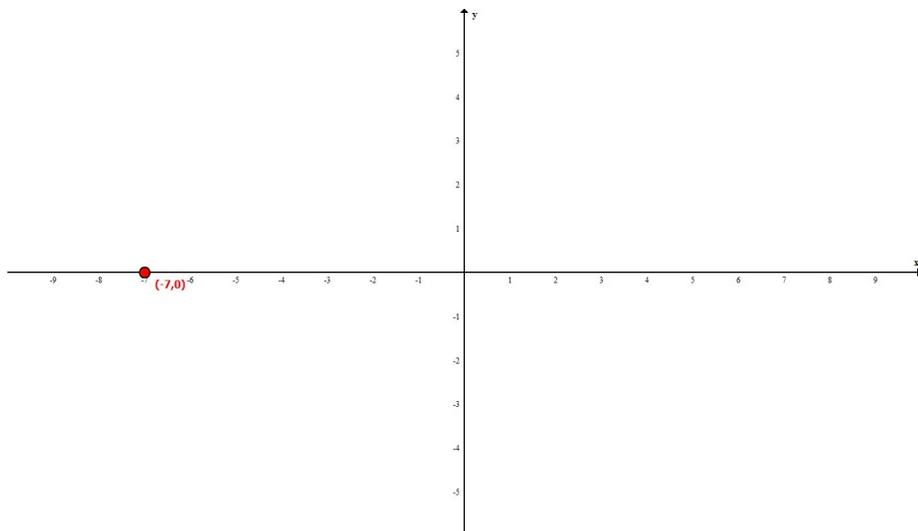
[Lines: Vertical, Horizontal, Parallel, Perpendicular]

MATH by Wilson
Your Personal Mathematics Trainer
MathByWilson.com

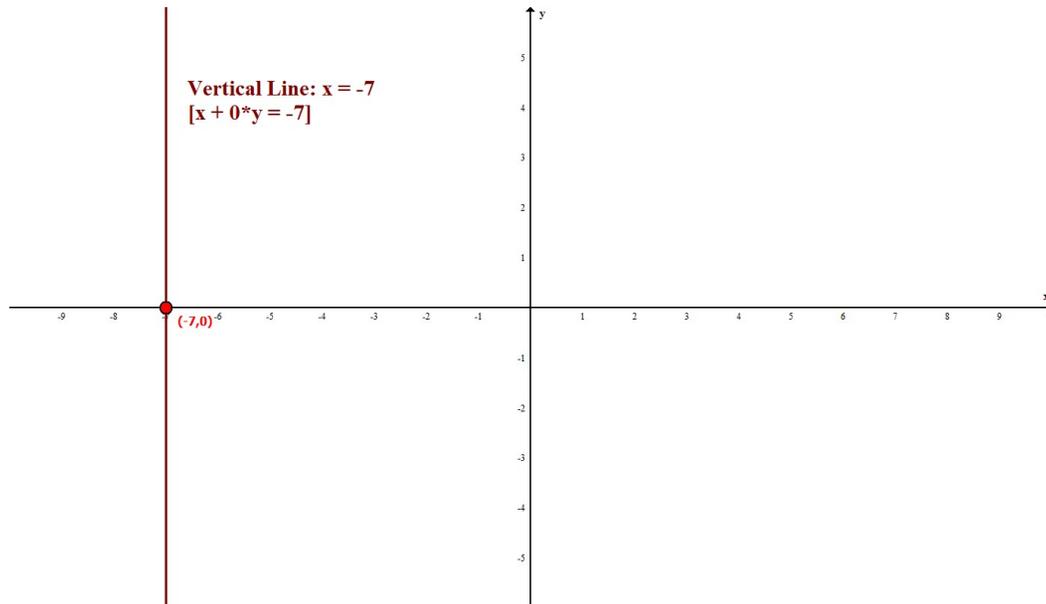
Vertical Lines: $x = \text{Number}$

Example 01: Draw the graph of $x = -7$.

Solution: If we are in one dimension, $x = -7$ is just a point on the Horizontal Number Line:



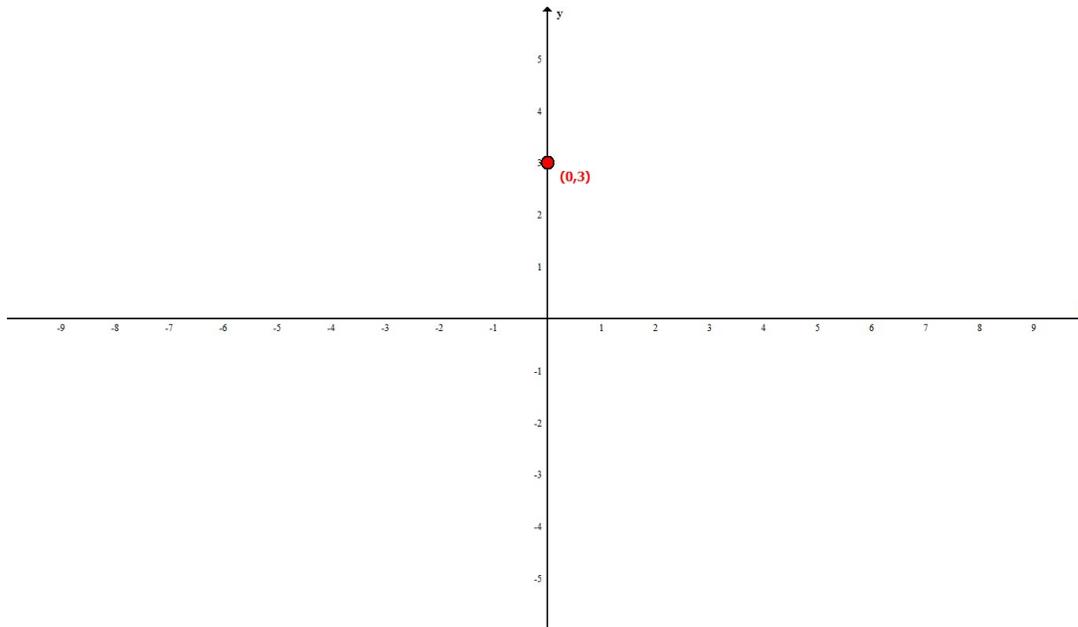
If we are in two dimensions, $x = -7$ can be written as $x + 0 \cdot y = -7$ so that points on its graph are $(-7, y)$ where y can be *any* number on the Vertical Number Line.



Horizontal Lines: $y = \text{Number}$

Example 02: Draw the graph of $y = 3$.

Solution: If we are in one dimension, $y = 3$ is just a point on the Vertical Number Line:



If we are in two dimensions, $y = 3$ can be written as $0 * x + y = 3$ so that points on its graph are $(x, 3)$ where x can be *any* number on the Horizontal Number Line.

Parallel Lines: $l_1 \parallel l_2 \Rightarrow m_1 = m_2$ obviously

Example 03: Given $l_1 : 4x + 3y = 12$, find the equation of the line l_2 parallel to l_1 that passes through the point $P(x_2, y_2) = P(-2, 3)$.

Solution: Solving $l_1 : 4x + 3y = 12$ for y yields $y = -\frac{4x}{3} + 4$ so that $m_2 = m_1 = -\frac{4}{3}$. Using the slope-intercept form $y = mx + b$, we obtain

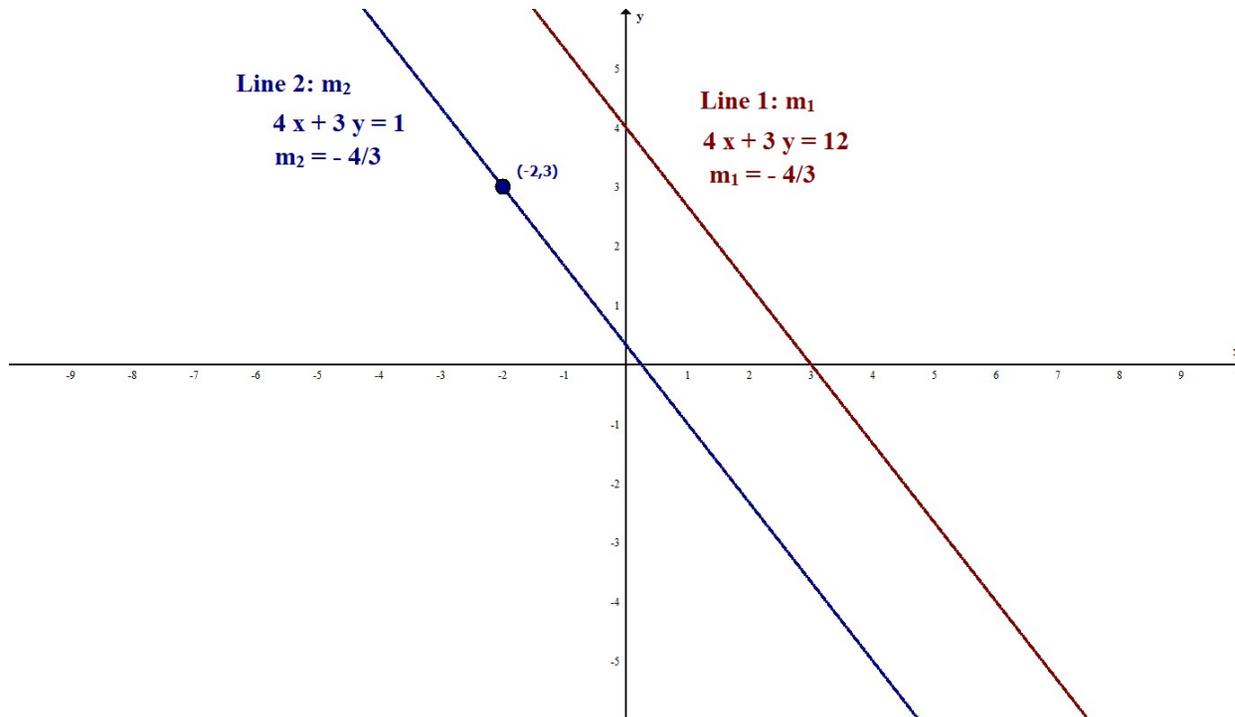
$$y_2 = m_2 x_2 + b$$

$$b = y_2 - m_2 x_2$$

$$= 3 - \left(-\frac{4}{3}(-2) \right)$$

$$= \frac{1}{3}$$

Thus $\ell_2 : y = -\frac{4x}{3} + \frac{1}{3}$. The graphs are shown below:



Perpendicular Lines: $l_1 \perp l_2 \Rightarrow m_1 * m_2 = -1$; this is NOT obvious but true: $m_2 = \frac{-1}{m_1}$ ($m_1 \neq 0$)

Example 04: Given $l_1 : 4x + 3y = 12$, find the equation of the line l_2 perpendicular to l_1 that passes through the point $P(x_2, y_2) = P(-2, 3)$.

Solution: Solving $l_1 : 4x + 3y = 12$ for y yields $y = -\frac{4x}{3} + 4$ so that $m_2 = -\frac{1}{m_1} = \frac{3}{4}$. Using the slope-intercept form

$y = mx + b$, we obtain

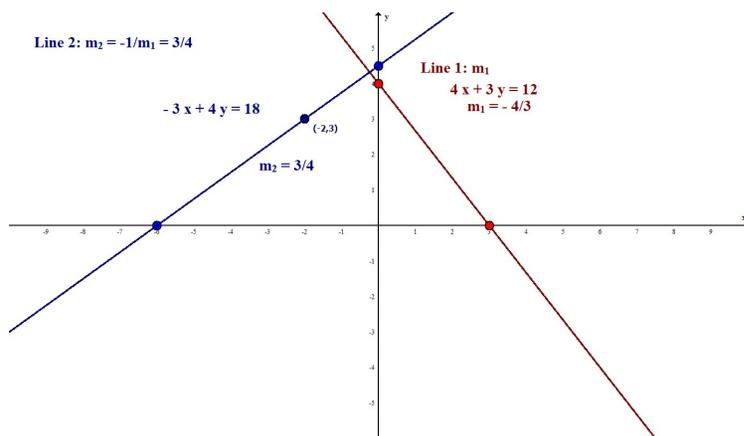
$$y_2 = m_2 x_2 + b$$

$$b = y_2 - m_2 x_2$$

$$= 3 - \left(\frac{3}{4}(-2) \right)$$

$$= 9/2$$

Thus $l_2 : y = \frac{3x}{4} + \frac{9}{2}$. The graphs are shown below:



Average Rate of Change: With respect to an Interval $[a,b]$

1. Linear Functions – Straight Lines ($y = f(x) = (\text{Slope}) * x + (\text{y-intercept})$)

$$\frac{\Delta y}{\Delta x} = \frac{\text{Change in y values}}{\text{Change in x values}} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a} = \text{Slope}$$

Example 05: Find the average rate of change for $y = f(x) = 3x + 4$ on $[2,5]$ & $[-1,3]$

Solution:

$[2,5]$: We have

$$\begin{aligned} \frac{f(b) - f(a)}{b - a} &= \frac{f(5) - f(2)}{5 - 2} = \frac{(15 + 4) - (6 + 4)}{3} \\ &= \frac{9}{3} = 3 \end{aligned}$$

$[-1,3]$: We have

$$\begin{aligned} \frac{f(b) - f(a)}{b - a} &= \frac{f(3) - f(-1)}{3 - (-1)} = \frac{(9 + 4) - (-3 + 4)}{4} \\ &= \frac{12}{4} = 3 \end{aligned}$$

Note: We have already noted that the slope of a straight line is independent of which points are selected.

2. Other Functions – NOT Straight Lines ($y = f(x) = \text{Formula}$)

$$\frac{\text{Change in y values}}{\text{Change in x values}} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

Example 06: Find the average rate of change for $y = f(x) = x^2 - 3x + 4$ on $[1,3]$ & $[-2,5]$.

Solution:

$[1,3]$: We have

$$\begin{aligned}\frac{f(b) - f(a)}{b - a} &= \frac{f(3) - f(1)}{3 - 1} = \frac{(9 - 9 + 4) - (1 - 3 + 4)}{2} \\ &= \frac{4}{2} = 2\end{aligned}$$

$[-2,5]$: We have

$$\begin{aligned}\frac{f(b) - f(a)}{b - a} &= \frac{f(5) - f(-2)}{5 - (-2)} = \frac{(25 - 15 + 4) - (4 + 6 + 4)}{7} \\ &= \frac{0}{7} = 0\end{aligned}$$

Note: For non-linear functions, the average rate of change may be dependent upon which two (2) points are selected.