FUNctions – Quadratic

$$\mathbf{y} = \mathbf{f}(\mathbf{x}) = \mathbf{ax}^2 + \mathbf{bx} + \mathbf{c} \; ; \; \mathbf{a} \neq 0$$

Five Point Method

$$\begin{bmatrix} \text{MATH by Wilson} \\ \text{Your Personal Mathematics Trainer} \\ \text{MathByWilson.com} \end{bmatrix}$$

With a linear function, we can obtain its graph by drawing a straight line through any two (2) points on the line. For a quadratic function, we can usually find five (5) special points and then draw the graph. The outline below shows how to find these points:

Given

$$\mathbf{y} = \mathbf{f}(\mathbf{x}) = \mathbf{ax}^2 + \mathbf{bx} + \mathbf{c} \; ; \; \mathbf{a} \neq 0$$

$\mathbf{a} =$ ________

$\mathbf{b} =$ ________

$\mathbf{c} =$ ________

find the following points and use them to draw the graph:

1. x-intercept points (usually 2 points)
2. y-intercept point
3. symmetry point (also called the “cheap point”)
4. vertex

a. **Domain**: Allowable x-values

$\textbf{Domain} = \mathbb{R}_{\mathbf{x}} = (-\infty, +\infty)_{\mathbf{x}}$

b. **POINTS #1 & #2:** Max of two x-intercept points

Set $\mathbf{D} = \mathbf{b}^2 - 4\mathbf{ac}$

(1) $\mathbf{D} < 0$: NO x-intercept point
(2) $\mathbf{D} = 0$: ONE x-intercept point
(3) $\mathbf{D} > 0$: TWO x-intercept points

x-intercepts POINTS(s): $\left(\frac{-\mathbf{b} \pm \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}, 0\right)$

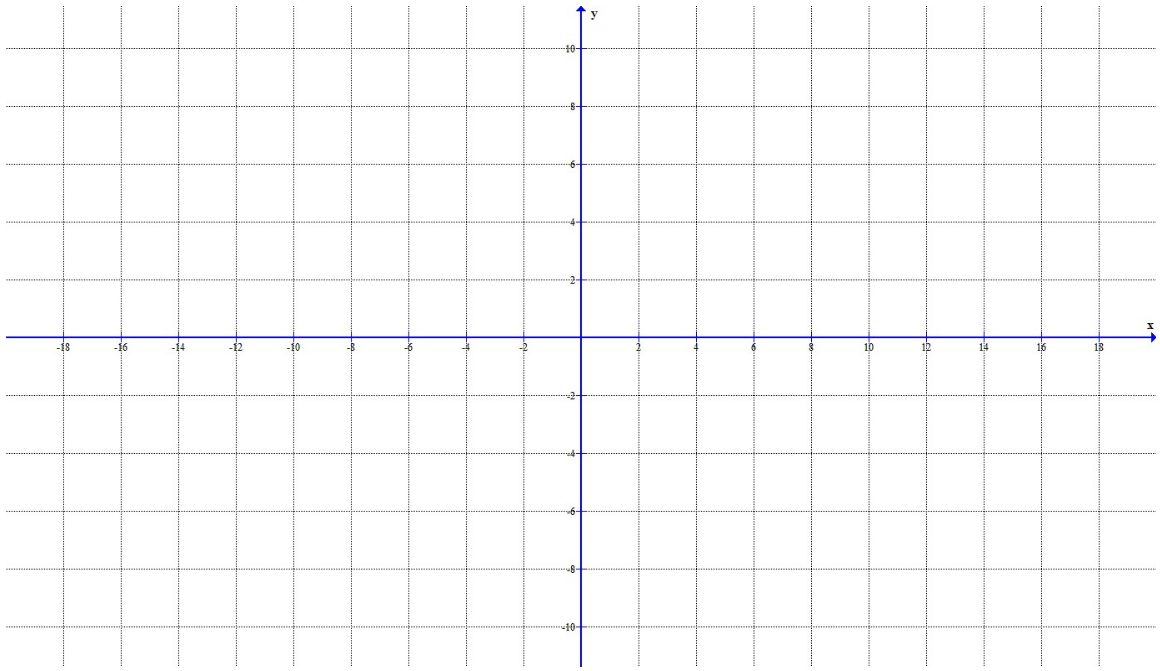
c. **POINT #3:**
y-intercept POINT: $(0, \mathbf{c})$

d. **POINT #4:**
Symmetry POINT: $\left(-\frac{\mathbf{b}}{\mathbf{a}}, \mathbf{c}\right)$ **;** also called “cheap point”

e. **POINT #5:**
Vertex POINT: $\left(-\frac{\mathbf{b}}{2\mathbf{a}}, \frac{4\mathbf{ac} - \mathbf{b}^2}{4\mathbf{a}}\right)$

Absolute Maximum point if $\mathbf{a} < 0$
Absolute Minimum point if $\mathbf{a} > 0$

f. **Graph:** PLOT the five (5) points (Max) and sketch the graph



g. **Positive:** The x-values where $\mathbf{y} > 0$

Negative: The x-values where $\mathbf{y} < 0$

Assuming two (2) x-intercept points, we have

(1) If $\mathbf{a} > 0$

$$\mathbf{Positive} = \left(-\infty, \frac{-\mathbf{b} - \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}\right] \cup \left[\frac{-\mathbf{b} + \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}, +\infty\right)$$

$$\mathbf{Negative} = \left[\frac{-\mathbf{b} - \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}, \frac{-\mathbf{b} + \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}\right]$$

(2) If $\mathbf{a} < 0$

$$\mathbf{Positive} = \left[\frac{-\mathbf{b} - \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}, \frac{-\mathbf{b} + \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}\right]$$

$$\mathbf{Negative} = \left(-\infty, \frac{-\mathbf{b} - \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}\right] \cup \left[\frac{-\mathbf{b} + \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}, +\infty\right)$$

h. **Range:** Allowable y-values

$$\textbf{Range} = \left(-\infty, \frac{4\mathbf{ac} - \mathbf{b}^2}{4\mathbf{a}}\right]_y \text{ if } \mathbf{a} < 0$$

$$\textbf{Range} = \left[\frac{4\mathbf{ac} - \mathbf{b}^2}{4\mathbf{a}}, +\infty\right)_y \text{ if } \mathbf{a} > 0$$