h(x) = A f(Bx + C) + D

Introduction

We have introduced a few Basic FUNctions y = f(x), together with their graphs and important properties. Now we construct new functions of the form

$$h(x) = A\,f(Bx + C) + D$$

where A, B, C, D are given parameters. Once we discuss the effects that the parameters have on f(x), we will be able to obtain the graph and properties of h(x). A one page h(x) Template is available which summarizes the effects the parameters have on f. However, our current purpose is to discuss the effect of each parameter analytically and geometrically. Consider

$$h(x) = A\,f(Bx + C) + D$$

First note that B and C are related to “x”, the domain. Since the orders of operations mandates multiplication before addition, we consider the B effect, then the C effect. Next we consider the range values

$$Ay + D$$

Which implies we consider the effect of A before the effect of D. Hence, the order of analysis is\

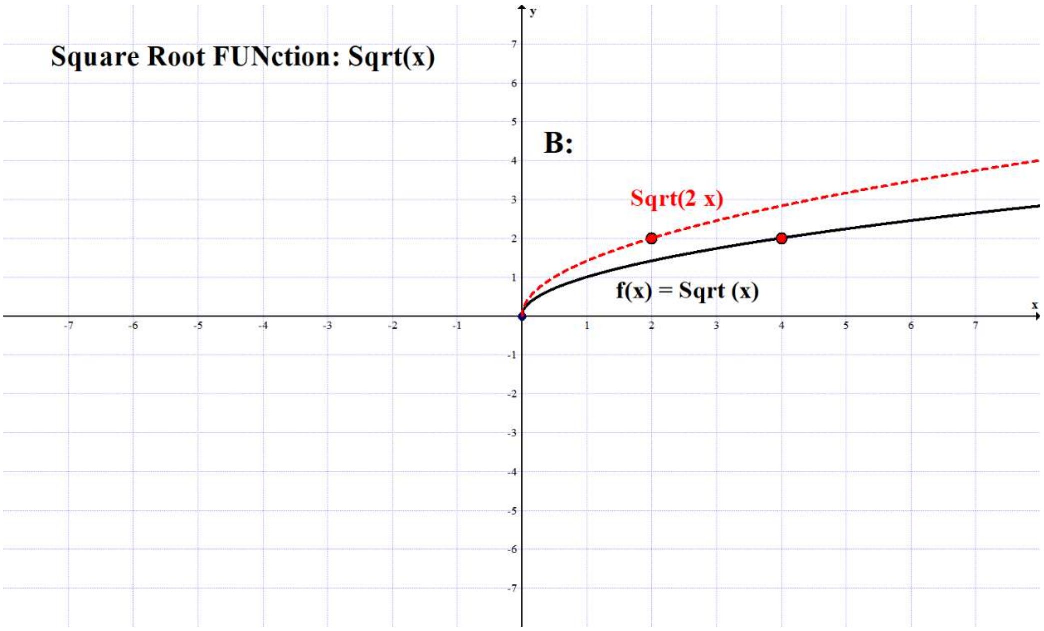
$$f(x) \rightarrow f(Bx) \rightarrow f(Bx + C) \rightarrow A\,f(Bx + C) \rightarrow A\,f(Bx + C0 + D = h(x)$$

Let’s now investigate the effects of B, C, A, D in order:

B: Domain Effect ; Horizontal ; f(x) → f(Bx)

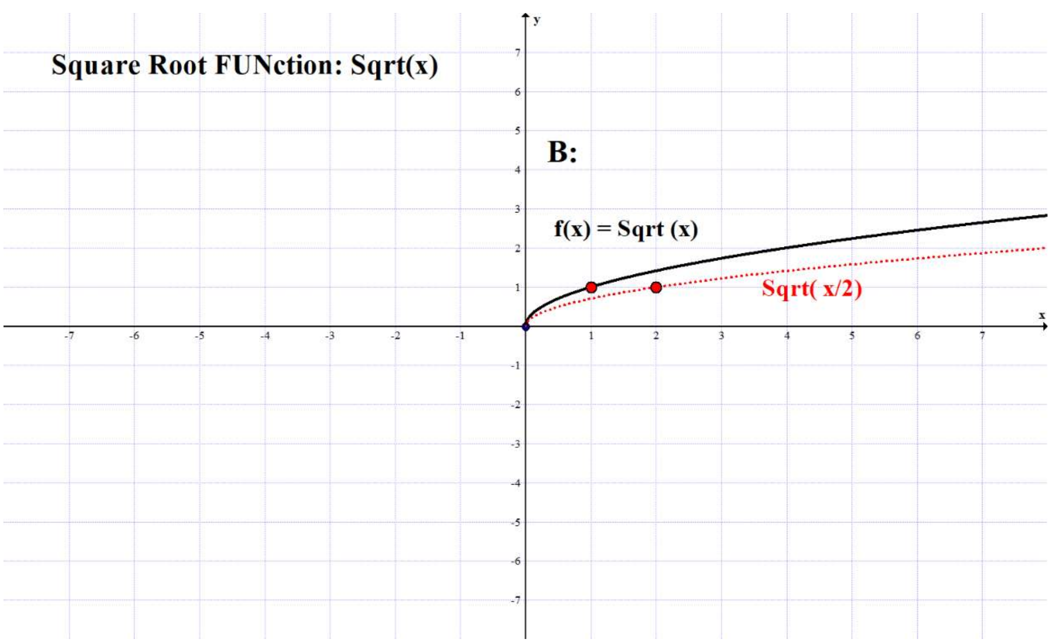
1. **Horizontal *Contraction*** when $|B| > 1$

$B = 2:$



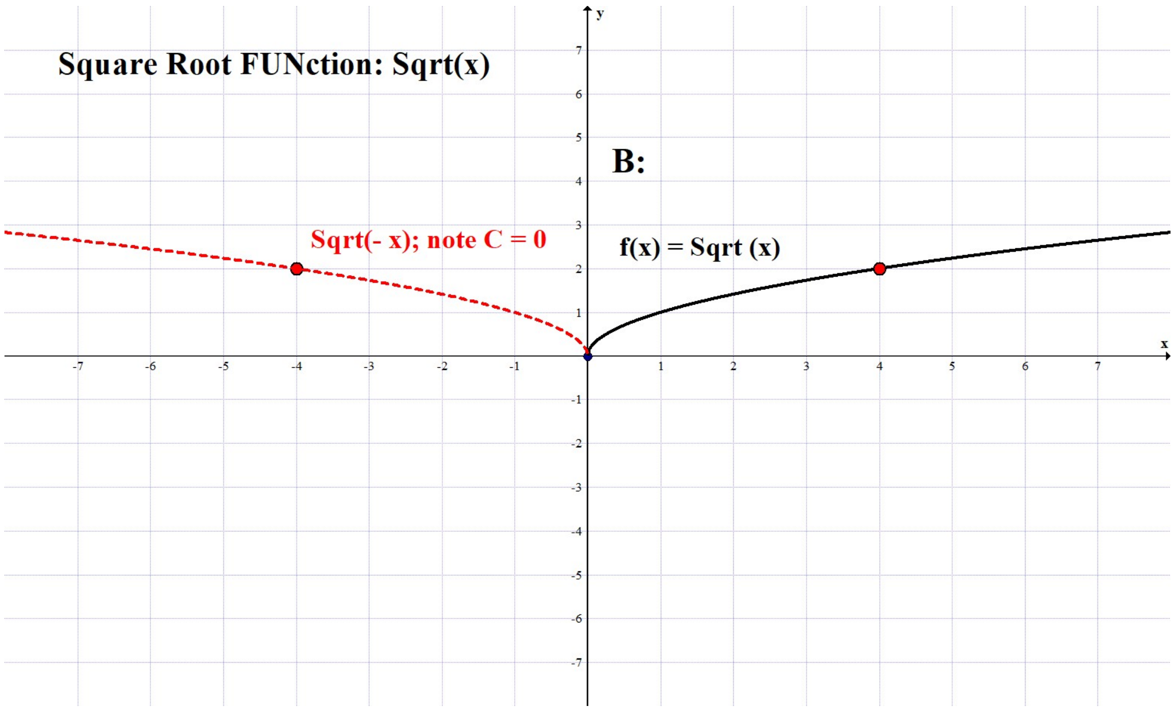
2. **Horizontal *Stretch*** when $0 < |B| < 1$:

$B = 1/2:$



3. ***Reflection*** in $\mathbf{x} = -\frac{\mathbf{C}}{\mathbf{B}}$ when $\mathrm{B} < 0$

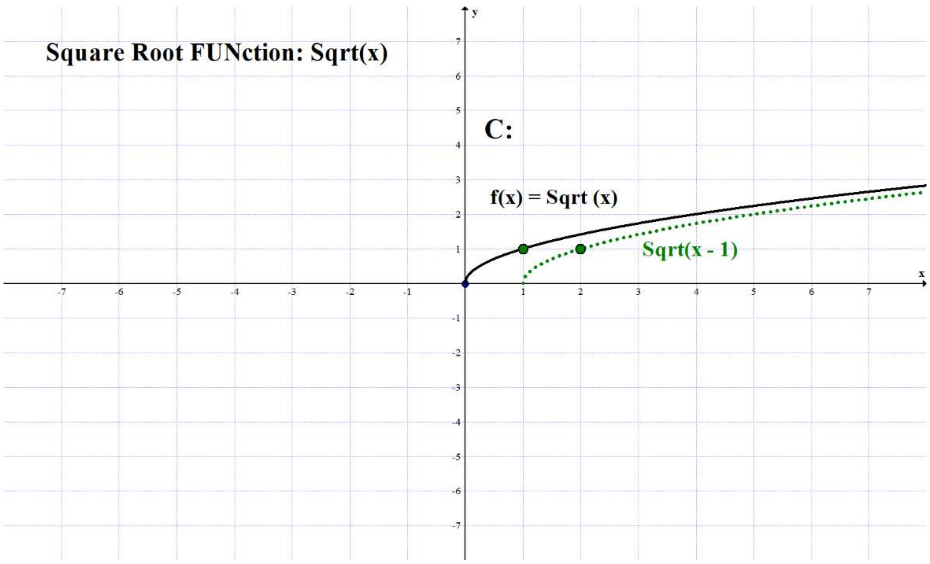
$\mathrm{B} = -1$; $\mathrm{C} = 0$:



C: Domain Effect ; Horizontal ; f(x) → f(Bx) → f(Bx + C)

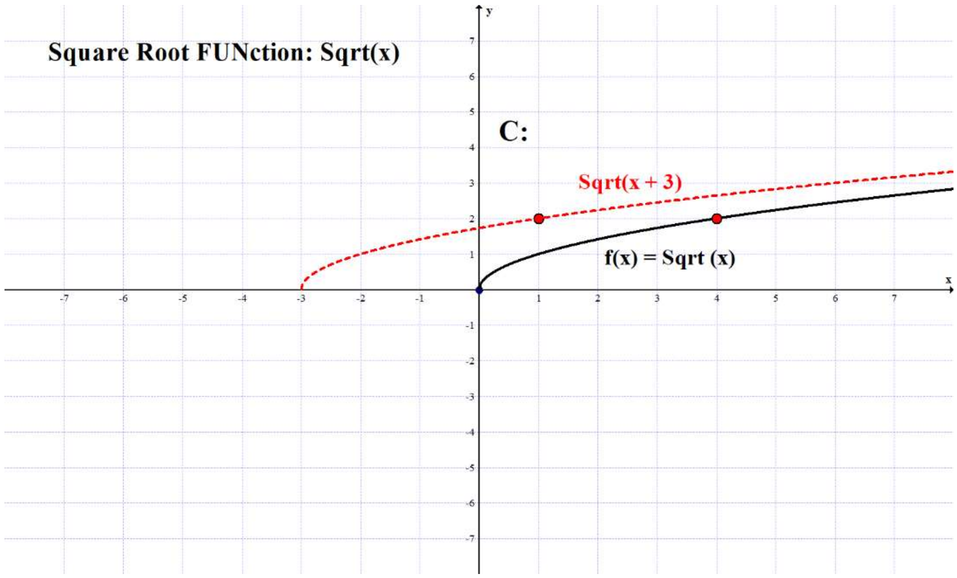
1. **Horizontal *Translation*** $\left|\frac{C}{B}\right|$ **units to the *right* when** $\frac{\mathbf{C}}{\mathbf{B}} < 0$

$B = 1 \; ; \; C = -1:$



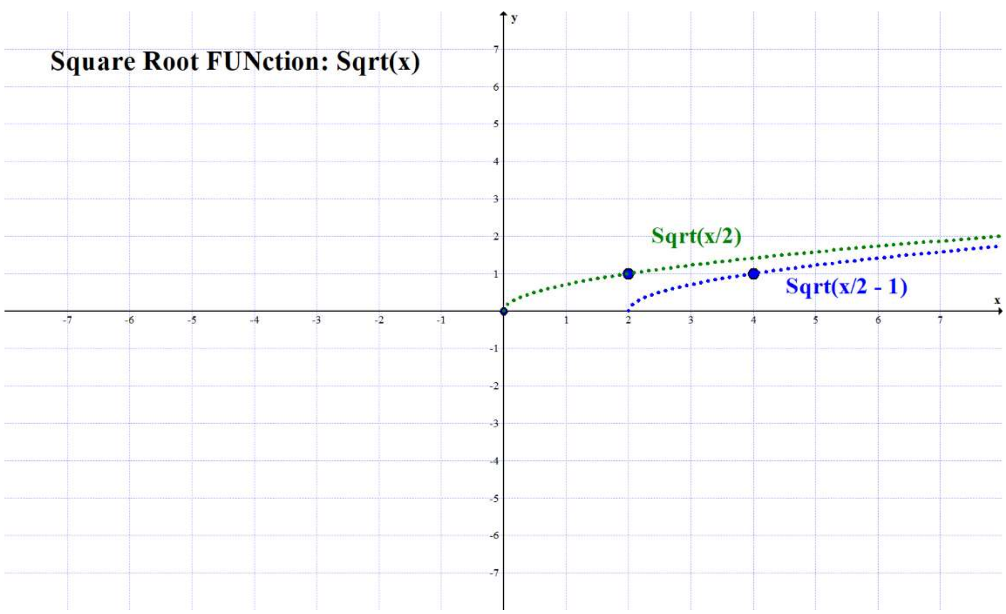
2. **Horizontal *Translation*** $\left|\frac{C}{B}\right|$ **units to the *left* when** $\frac{\mathbf{C}}{\mathbf{B}} > 0$

$B = 1 \; ; \; C = 3:$



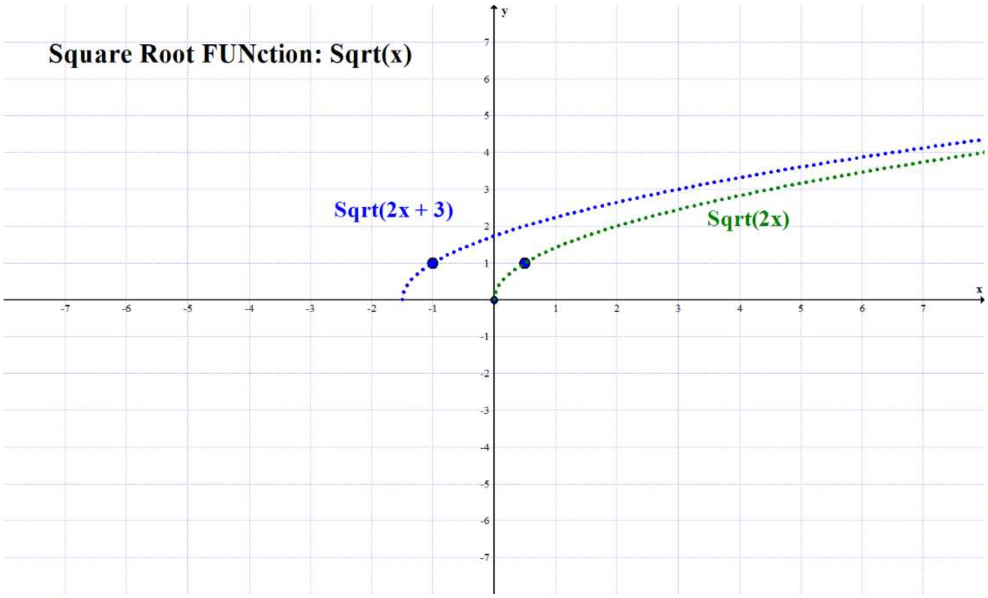
3. B = 1/2 ; C = - 1 :

$$\left|\frac{C}{B}\right| = 2$$



4. B = 2 ; C = 3 :

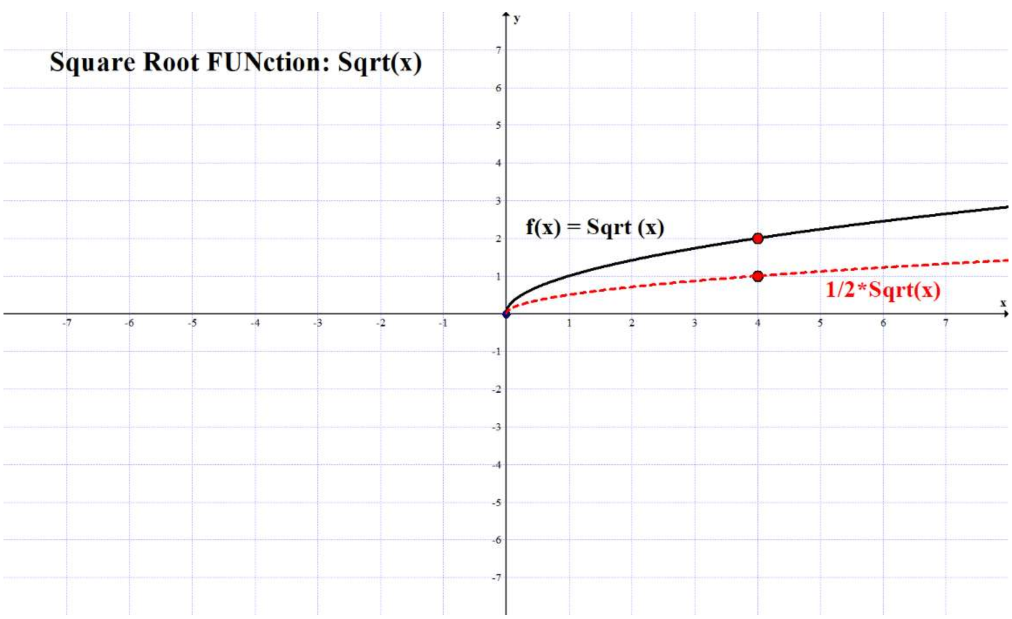
$$\left|\frac{C}{B}\right| = \frac{3}{2}$$



A: Range Effect ; Vertical ; f(x) → f(Bx) → f(Bx + C) → A f(Bx + C)

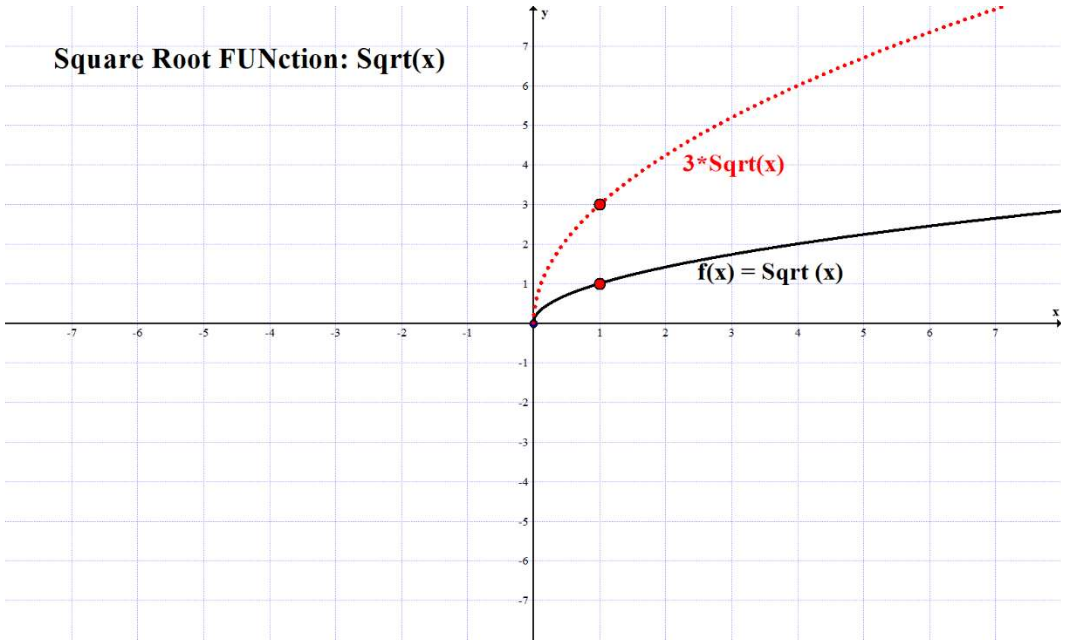
1. **Vertical *Contraction*** $0 < |A| < 1$ **; f(x) →A f(x)**

   A = 1 / 2 :



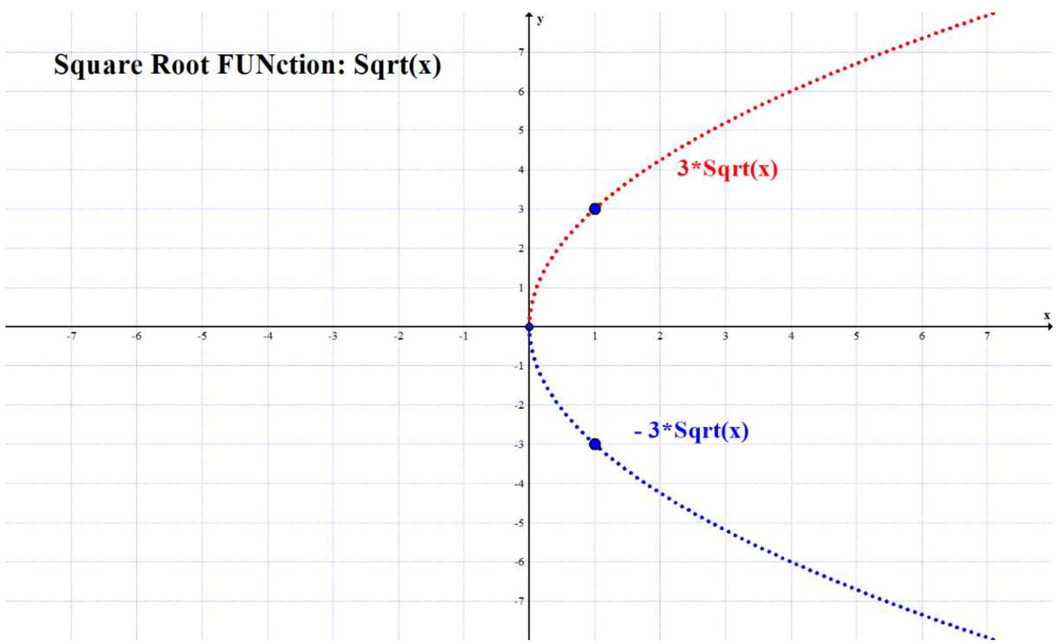
2. **Vertical *Stretch*** $|A| > 1$ **; f(x) →A f(x)**

   A = 3 :



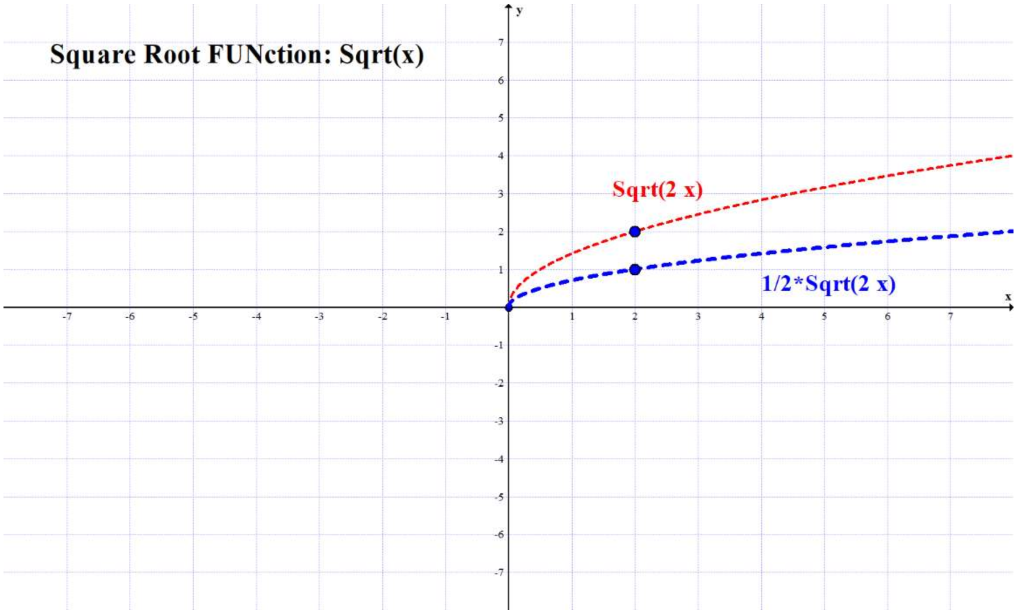
3. **If A < 0, the graph is reflected in the x-axis:**

A = - 3 :



4. **Whether you have f(x), f(Bx) or f(Bx + C), the A contracts or stretches its graph:**

B = 2 ; C = 0 ; A = 1/2 :



B = 4 ; C = 3 ; A = 2 :

B = 2 ; C = 0 :

5. ///////