Exponential and Logarithmic FUNction Properties

$\left[\begin{matrix}\text{MATH by Wilson}\\ \text{Your Personal Mathematics Trainer}\\ \text{MathByWilson.com}\end{matrix}\right]$

Note:

a. **Bases:** $0 < \mathbf{b} < 1$; $\mathbf{b} > 1$ $\quad \mathbf{e} \approx 2.71828828459045...$

b. **Assumes ALL quantities are defined, that is, real numbers (FYI: Can have complex numbers too)**

c. $\mathbf{b}^{\mathbf{x}}$ $[\mathbf{b} \wedge \mathbf{x}]$ $10^{\mathbf{x}}$ (Common Exponential) ; $\mathbf{e}^{\mathbf{x}}$ (Natural Exponential)

d. $\log_{\mathbf{b}} \mathbf{x} : \log_{10} \mathbf{x} = \log \mathbf{x}$ (Common Logarithm) ; $\log_{\mathbf{e}} \mathbf{x} = \ln \mathbf{x}$ (Natural Logarithm)

	Exponential (Exp)	Logarithmic (Log)
	(One Base)	
1	$\mathbf{b}^0 = 1$	$\log_{\mathbf{b}} 1 = 0$
2	$\mathbf{b}^1 = \mathbf{b}$	$\log_{\mathbf{b}} \mathbf{b} = 1$
3	$\mathbf{b}^{\mathbf{x}} = \mathbf{b}^{\mathbf{y}} \Leftrightarrow \mathbf{x} = \mathbf{y}$ (1-1)	$\log_{\mathbf{b}} \mathbf{x} = \log_{\mathbf{b}} \mathbf{y} \Leftrightarrow \mathbf{x} = \mathbf{y}$ (1-1)
4	$\log_{\mathbf{b}} \mathbf{b}^{\mathbf{x}} = \mathbf{x}$ (Inverse)	$\mathbf{b}^{\log_{\mathbf{b}} \mathbf{x}} = \mathbf{x}$ (Inverse)
5 ≠	$\log_{\mathbf{d}} \mathbf{b}^{\mathbf{x}} \neq \mathbf{x}$	$\mathbf{d}^{\log_{\mathbf{b}} \mathbf{x}} \neq \mathbf{x}$
6	$\mathbf{b}^{\mathbf{x}} = \mathbf{y} \Leftrightarrow \mathbf{x} = \log_{\mathbf{b}} \mathbf{y}$ **Trade exponential for logarithmic**	$\mathbf{x} = \log_{\mathbf{b}} \mathbf{y} \Leftrightarrow \mathbf{b}^{\mathbf{x}} = \mathbf{y}$ **Trade logarithmic for exponential**
7	$\left(\mathbf{b}^{\mathbf{x}}\right)^{\mathbf{y}} = \mathbf{b}^{\mathbf{x}*\mathbf{y}}$	$\log_{\mathbf{b}} \mathbf{x}^{\mathbf{y}} = \mathbf{y} * \log_{\mathbf{b}} \mathbf{x}$ **Trade exponential for multiplication or vice versa**
8 ≠	$\left(\mathbf{b}^{\mathbf{x}}\right)^{\mathbf{y}} \neq \mathbf{y} * \mathbf{b}^{\mathbf{x}}$	$\left(\log_{\mathbf{b}} \mathbf{x}\right)^{\mathbf{y}} \neq \mathbf{y} * \log_{\mathbf{b}} \mathbf{x}$
9	$\mathbf{b}^{\mathbf{x}} * \mathbf{b}^{\mathbf{y}} = \mathbf{b}^{\mathbf{x}+\mathbf{y}}$	$\log_{\mathbf{b}} (\mathbf{x} * \mathbf{y}) = \log_{\mathbf{b}} \mathbf{x} + \log_{\mathbf{b}} \mathbf{y}$ **Trade multiplication for addition or vice versa**
10 ≠	$\mathbf{b}^{\mathbf{x}+\mathbf{y}} \neq \mathbf{b}^{\mathbf{x}} + \mathbf{b}^{\mathbf{y}}$	$\log_{\mathbf{b}} (\mathbf{x} + \mathbf{y}) \neq \log_{\mathbf{b}} \mathbf{x} + \log_{\mathbf{b}} \mathbf{y}$
11 ≠	$\mathbf{b}^{\mathbf{x}-\mathbf{y}} \neq \mathbf{b}^{\mathbf{x}} - \mathbf{b}^{\mathbf{y}}$	$\log_{\mathbf{b}} (\mathbf{x} - \mathbf{y}) \neq \log_{\mathbf{b}} \mathbf{x} - \log_{\mathbf{b}} \mathbf{y}$

12	$\frac{b^x}{b^y} = b^{x-y} = \frac{1}{b^{y-x}}$	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$ **Trade division for subtraction or vice versa**
13≠	$\frac{b^x}{b^y} \neq \frac{x}{y}$	$\frac{\log_b x}{\log_b y} \neq \frac{x}{y}$
14		$\log_b x = \frac{\log_{10} x}{\log_{10} b} = \frac{\log_e x}{\log_e b}$ **Change of Base**
	(Two Bases)	
15	$(a * b)^x = a^x * b^x$	
16	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$	
17	$b^{-x} = \frac{1}{b^x}$; $\frac{1}{b^{-x}} = b^x$	
18	$\left(\frac{a}{b}\right)^{-x} = \left(\frac{b}{a}\right)^x$	
19≠	$(a+b)^x \neq a^x + b^x$	
20≠	$(a-b)^x \neq a^x - b^x$	
21	$a^x b^y = a^x b^y$	

Notes:

1. **Whatever $\square$ is , is called the argument of $b^{\square}$ or $\log_b \square$**
2. **You can replace "x" and "y" with something in the box:** $b^{\square}$ or $\log_b \square$
3. **Consider $x = \log_b y$. We can think of $\log_b y$ as the exponent to which to raise the base b to get y: $b^{\log_b y} = b^x = y$.**

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