Exponential and Logarithmic FUNction Properties

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Note:

a. **Bases:** $0 < \mathbf{b} < 1 \; ; \; \mathbf{b} > 1 \qquad \mathbf{e} \approx 2.71828828459045...$

b. **Assumes ALL quantities are defined, that is, real numbers (FYI: Can have complex numbers too)**

c. $\mathbf{b}^{\mathbf{x}}$ [b ^ x] $10^{\mathbf{x}}$ (Common Exponential) ; $\mathbf{e}^{\mathbf{x}}$ (Natural Exponential)

d. $\log_{\mathbf{b}} \mathbf{x}$: $\log_{10} \mathbf{x} = \log \mathbf{x}$ (Common Logarithm) ; $\log_{\mathbf{e}} \mathbf{x} = \ln \mathbf{x}$ (Natural Logarithm)

	Exponential (One Base)	Logarithmic
1	$\mathbf{b}^0 = 1$	$\log_{\mathbf{b}} 1 = 0$
	$10^0 = 1 \; ; \left(\frac{1}{2}\right)^0 = 1$	$\log_{10} 1 = 0 \; ; \log_{1/2} 1 = 0$
2	$\mathbf{b}^1 = \mathbf{b}$	$\log_{\mathbf{b}} \mathbf{b} = 1$
	$10^1 = 10 \; ; \left(\frac{1}{3}\right)^1 = \frac{1}{3}$	$\log_3 3 = 1; \log_{\mathbf{e}} \mathbf{e} = \ln \mathbf{e} = 1$
3	$\mathbf{b}^{\mathbf{x}} = \mathbf{b}^{\mathbf{y}} \Leftrightarrow \mathbf{x} = \mathbf{y}$ **(1-1)**	$\log_{\mathbf{b}} \mathbf{x} = \log_{\mathbf{b}} \mathbf{y} \Leftrightarrow \mathbf{x} = \mathbf{y}$ **(1-1)**
	$7^{\mathbf{x}} = 7^{\mathbf{y}} \Leftrightarrow \mathbf{x} = \mathbf{y}$	$\log_3 \mathbf{x} = \log_3 \mathbf{y} \Leftrightarrow \mathbf{x} = \mathbf{y}$
4	$\log_{\mathbf{b}} \mathbf{b}^{\mathbf{x}} = \mathbf{x}$ (Inverse)	$\mathbf{b}^{\log_{\mathbf{b}} \mathbf{x}} = \mathbf{x}$ (Inverse)
	$\log_4 4^{\mathbf{x}} = \mathbf{x}$	$2^{\log_2 \mathbf{x}} = \mathbf{x}$
5	$\mathbf{b}^{\mathbf{x}} = \mathbf{y} \Leftrightarrow \mathbf{x} = \log_{\mathbf{b}} \mathbf{y}$ Trade exponential for logarithmic	$\mathbf{x} = \log_{\mathbf{b}} \mathbf{y} \Leftrightarrow \mathbf{b}^{\mathbf{x}} = \mathbf{y}$ Trade logarithmic for exponential
	$\left(\frac{1}{2}\right)^{\mathbf{x}} = \mathbf{y} \Leftrightarrow \mathbf{x} = \log_{1/2} \mathbf{y}$	$\mathbf{x} = \log_8 \mathbf{y} \Leftrightarrow 8^{\mathbf{x}} = \mathbf{y}$
6	$\left(\mathbf{b}^{\mathbf{x}}\right)^{\mathbf{y}} = \mathbf{b}^{\mathbf{x}*\mathbf{y}}$	$\log_{\mathbf{b}} \mathbf{x}^{\mathbf{y}} = \mathbf{y} * \log_{\mathbf{b}} \mathbf{x}$ Trade exponential for multiplication
	$\left(4^2\right)^3 = 4096 = 4^{2*3}$	$\log_2 8^3 = 9 = 3 * \log_2 8$

7 ≠	$\left(b^{x}\right)^{y} \neq y * b^{x}$	$\left(\log_b x\right)^{y} \neq y * \log_b x$
	$\left(2^{2.1}\right)^{3.25} = 113.378 \neq 13.933 = 3.25 * 2^{2.1}$	$\left(\log_3 9\right)^{4} = 16 \neq 8 = 4 * \log_3 9$
8	$b^{x} * b^{y} = b^{x+y}$	$\log_b\left(x * y\right) = \log_b x + \log_b y$ **Trade multiplication for addition**
	$2^{3} * 2^{4} = 128 = 2^{3+4}$	$\log_3\left(3 * 9\right) = \log_3 3 + \log_3 9$
9 ≠	$b^{x+y} \neq b^{x} + b^{y}$	$\log_b\left(x + y\right) \neq \log_b x + \log_b y$
	$2^{3+4} = 128 \neq 24 = 2^{3} + 2^{4}$	
10 ≠	$b^{x-y} \neq b^{x} - b^{y}$	$\log_b\left(x - y\right) \neq \log_b x - \log_b y$
	$2^{3-4} = \frac{1}{2} \neq -8 = 2^{3} - 2^{4}$	$\log_2\left(8-4\right) = 2 \neq 1 = \log_2 8 - \log_2 4$
11	$\frac{b^{x}}{b^{y}} = b^{x-y} = \frac{1}{b^{y-x}}$	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$ **Trade division for subtraction**
	$\frac{4^{3}}{4^{2}} = 4 = 4^{3-2} = 4 = \frac{1}{4^{2-3}}$	$\log_4\left(\frac{16}{4}\right) = 1 = \log_4 16 - \log_4 4$
12 ≠	$\frac{b^{x}}{b^{y}} \neq \frac{x}{y}$	$\frac{\log_b x}{\log_b y} \neq \frac{x}{y}$
	$\frac{3^{4}}{3^{2}} = 9 \neq 2 = \frac{4}{2}$	$\frac{\log_6 216}{\log_6 36} = \frac{3}{2} \neq 6 = \frac{216}{36}$
13		$\log_b x = \frac{\log_{10} x}{\log_{10} b} = \frac{\log_e x}{\log_e b}$ **Change of Base**
		$\log_3 81 = 4 = \frac{\log_{10} 81}{\log_{10} 3} = 4 = \frac{\log_e 81}{\log_e 3}$
	(Two Bases)	
14	$\left(a * b\right)^{x} = a^{x} * b^{x}$	
	$\left(2 * 5\right)^{3} = 1000 = 2^{3} * 5^{3}$	

15	$\left(\frac{\mathbf{a}}{\mathbf{b}}\right)^{\mathbf{x}} = \frac{\mathbf{a}^{\mathbf{x}}}{\mathbf{b}^{\mathbf{x}}}$	
	$\left(\frac{10}{2}\right)^{4} = 625 = \frac{10^{4}}{2^{4}}$	
16	$\mathbf{b}^{-\mathbf{x}} = \frac{1}{\mathbf{b}^{\mathbf{x}}}\,;\, \frac{1}{\mathbf{b}^{-\mathbf{x}}} = \mathbf{b}^{\mathbf{x}}$	
	$4^{-3} = \frac{1}{64} = \frac{1}{4^{3}}\,;\, \frac{1}{4^{-3}} = 64 = 4^{3}$	
17	$\left(\frac{\mathbf{a}}{\mathbf{b}}\right)^{-\mathbf{x}} = \left(\frac{\mathbf{b}}{\mathbf{a}}\right)^{\mathbf{x}}$	
	$\left(\frac{4}{2}\right)^{-3} = \frac{1}{8} = \left(\frac{2}{4}\right)^{3}$	
18 $\neq$	$(\mathbf{a}+\mathbf{b})^{\mathbf{x}} \neq \mathbf{a}^{\mathbf{x}} + \mathbf{b}^{\mathbf{x}}$	
	$(2+3)^{4} = 625 \neq 97 = 2^{4} + 3^{4}$	
19 $\neq$	$(\mathbf{a}-\mathbf{b})^{\mathbf{x}} \neq \mathbf{a}^{\mathbf{x}} - \mathbf{b}^{\mathbf{x}}$	
	$(4-2)^{3} = 8 \neq 56 = 4^{3} - 2^{3}$	
20	$\mathbf{a}^{\mathbf{x}}\mathbf{b}^{\mathbf{y}} = \mathbf{a}^{\mathbf{x}}\mathbf{b}^{\mathbf{y}}$	
	$2^{3}5^{2} = 200 = 2^{3}5^{2}$	

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