

Inverse of a Matrix
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Important Fact:

Assume that A is an n by n matrix with $\det(A) = |A| \neq 0$. Then there exist a matrix denoted A^{-1} and called the inverse of A such that

1. $A A^{-1} = I_n$
2. $A^{-1} A = I_n$

where I_n is the n by n Identity Matrix. We will see the usefulness of the inverse, when it exists, soon. To find the inverse we use row operations to transform $[A|I_n]$ to $[I_n|A^{-1}]$

Example 01: 2x2

Find the inverse of

$$A = \begin{bmatrix} 5 & -2 \\ 3 & 4 \end{bmatrix}$$

Solution :

Goal: Transform $\left[\begin{array}{cc|cc} 5 & -2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array} \right]$ into $\left[\begin{array}{cc|cc} 1 & 0 & A \text{Inverse} \\ 0 & 1 & A^{-1} \end{array} \right]$ using the Row Operations:

$$\frac{1}{5} * R_1 \rightarrow R_1 : \left[\begin{array}{cc|cc} 1 & \frac{-2}{5} & \frac{1}{5} & 0 \\ 3 & 4 & 0 & 1 \end{array} \right] \text{ <Multiply Row 1 by "1/5" and store result in Row 1>}$$

$$-3 * R_1 + R_2 \rightarrow R_2 : \left[\begin{array}{cc|cc} 1 & \frac{-2}{5} & \frac{1}{5} & 0 \\ 0 & \frac{26}{5} & \frac{-3}{5} & 1 \end{array} \right] \text{ <Multiply Row 1 by "-3" then add Row 2 and store}$$

the result in Row 2>

$$\frac{5}{26} * R_2 \rightarrow R_2 : \left[\begin{array}{cc|cc} 1 & -\frac{2}{5} & \frac{1}{5} & 0 \\ 0 & 1 & -\frac{3}{26} & \frac{5}{26} \end{array} \right] \text{ <Multiply Row 2 by "5/26" and store result in Row 2>$$

$$\frac{2}{5} * R_2 + R_1 \rightarrow R_1 : \left[\begin{array}{cc|cc} 1 & 0 & \frac{4}{26} & \frac{2}{26} \\ 0 & 1 & -\frac{3}{26} & \frac{5}{26} \end{array} \right] \text{ <Multiply Row 2 by "2/5" then add Row 1 and store}$$

result is Row 1

$$\text{Hence } A^{-1} = \begin{bmatrix} \frac{4}{26} & \frac{2}{26} \\ -\frac{3}{26} & \frac{5}{26} \end{bmatrix} = \frac{1}{26} \begin{bmatrix} 4 & 2 \\ -3 & 5 \end{bmatrix}$$

Note: $A A^{-1} = A^{-1} A = I_2$

Example 02: 3x3

Find the inverse of

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ 4 & 1 & 2 \end{bmatrix}$$

Solution:

$$\text{Goal: Transform } \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 2 & 3 & -1 & 0 & 1 & 0 \\ 4 & 1 & 2 & 0 & 0 & 1 \end{array} \right] \text{ into } \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & & & \\ 0 & 1 & 0 & & & \\ 0 & 0 & 1 & & & \end{array} \right] A^{-1} :$$

$-2 * R_1 + R_2 \rightarrow R_2$ <Multiply Row 1 by "-2" then add Row 2 and store the result in Row 2>

$-4 * R_1 + R_3 \rightarrow R_3$ <Multiple Row 1 by "-4" then add Row 3 and store the result in Row 3>

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 7 & -7 & -2 & 1 & 0 \\ 0 & 9 & -10 & -4 & 0 & 1 \end{array} \right]$$

$\frac{1}{7} * R_2 \rightarrow R_2$ <Multiply Row 2 by "1/7" and store the result in Row 2>

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -\frac{2}{7} & \frac{1}{7} & 0 \\ 0 & 9 & -10 & -4 & 0 & 1 \end{array} \right]$$

$2 * R_2 + R_1 \rightarrow R_1$ <Multiply Row 2 by "2" then add Row 1 and store the result in Row 1>

$-9R_2 + R_3 \rightarrow R_3$ <Multiple Row 2 by "-9" then add Row 3 and store the result in Row 3>

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & \frac{3}{7} & \frac{2}{7} & 0 \\ 0 & 1 & -1 & -\frac{2}{7} & \frac{1}{7} & 0 \\ 0 & 0 & -1 & -\frac{10}{7} & -\frac{9}{7} & 1 \end{array} \right]$$

$-1 * R_3 \rightarrow R_3$ <Row 3 by "-1" and store result in Row 3>

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & \frac{3}{7} & \frac{2}{7} & 0 \\ 0 & 1 & -1 & -\frac{2}{7} & \frac{1}{7} & 0 \\ 0 & 0 & 1 & \frac{10}{7} & \frac{9}{7} & -1 \end{array} \right]$$

$-1 * R_3 + R_1 \rightarrow R_1$ <Multiply Row 3 by "-1" the add Row 1 and store the result in Row 1>

$1 * R_3 + R_2 \rightarrow R_2$ <Add Row 3 to Row 2 and store result in Row 2>

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -1 & 1 \\ 0 & 1 & 0 & \frac{8}{7} & \frac{10}{7} & -1 \\ 0 & 0 & 1 & \frac{10}{7} & \frac{9}{7} & -1 \end{array} \right]$$

$$\text{Hence } A^{-1} = \begin{bmatrix} -1 & -1 & 1 \\ \frac{8}{7} & \frac{10}{7} & -1 \\ \frac{10}{7} & \frac{9}{7} & -1 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} -7 & -7 & 7 \\ 8 & 10 & -7 \\ 10 & 9 & -7 \end{bmatrix}$$

Note: $A A^{-1} = A^{-1} A = I_3$

ERROR: undefined
OFFENDING COMMAND:

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