

Systems of Linear Equations

3x3 Gauss-Jordan Matrix

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3x3:

$$p_1 : x - 2y + 3z = 14$$

$$p_2 : 2x + 3y - z = -7$$

$$p_3 : 4x + y + 2z = 7$$

$$\text{Solution: } (x, y, z) = (?, ?, ?)$$

Gauss-Jordan Method (A Matrix Method):

$$p_1 : x - 2y + 3z = 14$$

$$p_2 : 2x + 3y - z = -7$$

$$p_3 : 4x + y + 2z = 7$$

Augmented Matrix

$\begin{array}{ccc c} x & y & z & \\ \hline 1 & -2 & 3 & 14 \\ 2 & 3 & -1 & -7 \\ 4 & 1 & 2 & 7 \end{array}$	$\text{Goal: } \begin{array}{ccc c} 1 & 0 & 0 & x \text{ sol} \\ 0 & 1 & 0 & y \text{ sol} \\ 0 & 0 & 1 & z \text{ sol} \end{array}$
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To transform $\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ 4 & 1 & 2 \end{bmatrix}$ into $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, we should do the steps in the "wisdom order":

1. Make $a = 1$
2. Make $b = 0$
3. Make $c = 0$
4. Make $e = 1$
5. Make $d = 0$
6. Make $f = 0$
7. Make $i = 1$
8. Make $g = 0$
9. Make $h = 0$

$-2 * R_1 + R_2 \rightarrow R_2$ <Multiply Row 1 by "-2" then add Row 2 and store the result in Row 2>

$-4 * R_1 + R_3 \rightarrow R_3$ <Multiply Row 1 by "-4" then add Row 3 and store the result in Row 3>

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 14 \\ 0 & 7 & -7 & -35 \\ 0 & 9 & -10 & -49 \end{array} \right]$$

$\frac{1}{7} * R_2 \rightarrow R_2$ <Multiply Row 2 by "1/7" and store the result in Row 2>

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 14 \\ 0 & 1 & -1 & -5 \\ 0 & 9 & -10 & -49 \end{array} \right]$$

$2 * R_2 + R_1 \rightarrow R_1$ <Multiply Row 2 by "2" then add Row 1 and store the result in Row 1>

$-9R_2 + R_3 \rightarrow R_3$ <Multiple Row 2 by "-9" then add Row 3 and store the result in Row 3>

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & -1 & -4 \end{array} \right]$$

$-1 * R_3 \rightarrow R_3$ <Multiple Row 3 by "-1" and store result in Row 3>

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 1 & 4 \end{array} \right]$$

$-1 * R_3 + R_1 \rightarrow R_1$ <Multiply Row 3 by "-1" the add Row 1 and store the result in Row 1>

$1 * R_3 + R_2 \rightarrow R_2$ <Add Row 3 to Row 2 and store result in Row 2>

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 4 \end{array} \right]$$

The solution is in the last column: Solution: $(x, y, z) = (0, -1, 4)$