Systems of Linear Equations
(n by n: "n" equations with "n" unknowns)
Inverse Solutions

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In "rookie algebra", we solved linear equations like

$$3x = 5$$
$$x = \frac{5}{3} = 3^{-1} * 5$$

However, in general, we wrote

$$ax = b$$
$$x = \frac{b}{a} = a^{-1} * b \quad ; a \neq 0$$

If you are wondering why I wrote $x = a^{-1} * b$ as the solution, it is because certain n by n linear systems have solutions that can be written the same way.

Example 2x2:

Consider the 2 by 2 linear system we previously solved multiple ways:

$$l_1 : 5x - 2y = 19$$
$$l_2 : 3x + 4y = 1$$
$$Solution : (x, y) = (?, ?)$$

Setting $A = \begin{bmatrix} 5 & -2 \\ 3 & 4 \end{bmatrix}$; $B = \begin{bmatrix} 19 \\ 1 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \end{bmatrix}$, we see that $AX = B$ is equivalent to the given system:

$$AX = \begin{bmatrix} 5 & -2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5x - 2y \\ 3x + 4y \end{bmatrix} = \begin{bmatrix} 19 \\ 1 \end{bmatrix} = B$$

Since the $\det(A) \neq 0$ & $A^{-1} = \begin{bmatrix} \frac{4}{26} & \frac{2}{26} \\ -\frac{3}{26} & \frac{5}{26} \end{bmatrix}$ (previously calculated), we have

$$X = I\ X = A^{-1}A\ X = A^{-1}\ B = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

which yields the same solution as before.

Example 3x3:

Consider the 3 by 3 linear system we previously solved multiple ways:

$$\begin{aligned} &p_1 : x - 2y + 3z = 14 \\ &p_2 : 2x + 3y - z = -7 \\ &p_3 : 4x + y + 2z = 7 \\ &Solution : (x, y, z) = (?,?,?) \end{aligned}$$

Setting $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ 4 & 1 & 2 \end{bmatrix}; B = \begin{bmatrix} 14 \\ -7 \\ 7 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, we see that $AX = B$ is equivalent to the given system:

$$AX = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ 4 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x - 2y + 3z \\ 2x + 3y - z \\ 4x + y + 2z \end{bmatrix} = \begin{bmatrix} 14 \\ -7 \\ 7 \end{bmatrix} = B$$

Since the $\det(A) \neq 0$ & $A^{-1} = \begin{bmatrix} -1 & -1 & 1 \\ \frac{8}{7} & \frac{10}{7} & -1 \\ \frac{10}{7} & \frac{9}{7} & -1 \end{bmatrix}$ (previously calculated), we have

$$X = I\ X = A^{-1}A\ X = A^{-1}\ B = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

which yields the same solution as before.

