Matrix Operations

$$\begin{bmatrix} \text{MATH by Wilson} \\ \text{Your Personal Mathematics Trainer} \\ \text{MathByWilson.com} \end{bmatrix}$$

1. **Definition:** A **matrix** A has the form

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ . & & & . \\ . & & & . \\ . & & & . \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix} = \left[a_{ij}\right]_{\substack{i=1,\dots,n \\ j=1,\dots,m}} = \left[a_{ij}\right] \quad ;\text{n rows; m columns}$$

2. **Sum:** Component wise

$S = A + B$; $\left[s_{ij}\right] \equiv \left[a_{ij} + b_{ij}\right]$; A and B MUST be the same size: n by m

Example 01:

$$A + B = \begin{bmatrix} 3 & -4 & 5 & 7 \\ -2 & 0 & 3 & -5 \\ 6 & 2 & -2 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 3 & -2 & 1 \\ 3 & 1 & 0 & -1 \\ -7 & 4 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 3 & 8 \\ 1 & 1 & 3 & -6 \\ -1 & 6 & 0 & 6 \end{bmatrix} = S$$

Note: $s_{12} = a_{12} + b_{12} = (-4) + (3) = -1$; verify the remaining entries.

3. **Difference:** Component wise

$D = A - B$; $\left[d_{ij}\right] \equiv \left[a_{ij} - b_{ij}\right]$; A and B MUST be the same size: n by m

Example 02:

$$A - B = \begin{bmatrix} 3 & -4 & 5 & 7 \\ -2 & 0 & 3 & -5 \\ 6 & 2 & -2 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 3 & -2 & 1 \\ 3 & 1 & 0 & -1 \\ -7 & 4 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 5 & -7 & 7 & 6 \\ -5 & -1 & 3 & -4 \\ 13 & -2 & -4 & -4 \end{bmatrix} = D$$

Note: $s_{34} = a_{34} - b_{34} = (1) - (5) = -4$; verify the remaining entries.

4. Scalar Product - Constant c times a matrix A: Component wise

$$cA = \left[c\, a_{ij} \right]$$

Example 03:

$$-3A = -3 \begin{bmatrix} 3 & -4 & 5 & 7 \\ -2 & 0 & 3 & -5 \\ 6 & 2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -9 & 12 & -15 & -21 \\ 6 & 0 & -9 & 15 \\ -18 & -6 & 6 & -3 \end{bmatrix}$$

Note: $-3\, a_{23} = -3\,(3) = -9$; verify the remaining entries.

5. Product: NOT component wise

$P = AB$; $\left[p_{ij} \right] \equiv \left[\sum_{k=1}^{n} a_{ik}\, b_{kj} \right]$; Number of columns in A must be the same as the number of rows in B: A (n by m) and B (m by p) so that P is n by p.

Example 04:

$$AB = \begin{bmatrix} 3 & -4 & 5 & 7 \\ -2 & 0 & 3 & -5 \\ 6 & 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 4 & -3 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -12 \\ 25 & -17 \\ -5 & 6 \end{bmatrix} = \text{P}$$

Note: $p_{21} = (-2)(1) + (0)(0) + (3)(4) + (-5)(-3) = 25$; verify the remaining entries.

Rotate Row	Rewrite column	Multiply
−2	1	− 2
0	0	0
3	4	12
−5	− 3	15 Add
		25

6. **Division –** Not directly ; sometimes indirectly with “inverses”

ERROR: undefined
OFFENDING COMMAND:

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