FUNctions – Polynomials
Lower Degree

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Polynomials:

$\mathbf{y} = \mathbf{f}(\mathbf{x}) = \mathbf{a_n x^n} + \mathbf{a_{n-1} x^{n-1}} + \ldots + \mathbf{a_3 x^3} + \mathbf{a_2 x^2} + \mathbf{a_1 x} + \mathbf{a_0}$; where **n** is non-negative integer & $\mathbf{a_n}, \mathbf{a_{n-1}}, \ldots, \mathbf{a_2}, \mathbf{a_1}, \mathbf{a_0} \in \mathbb{R}$ (given)

Example: $\mathbf{y} = \mathbf{f}(\mathbf{x}) = 6\mathbf{x}^3 + 13\mathbf{x}^2 - 13\mathbf{x} - 30$; **Deg f** $= 3$

Note:

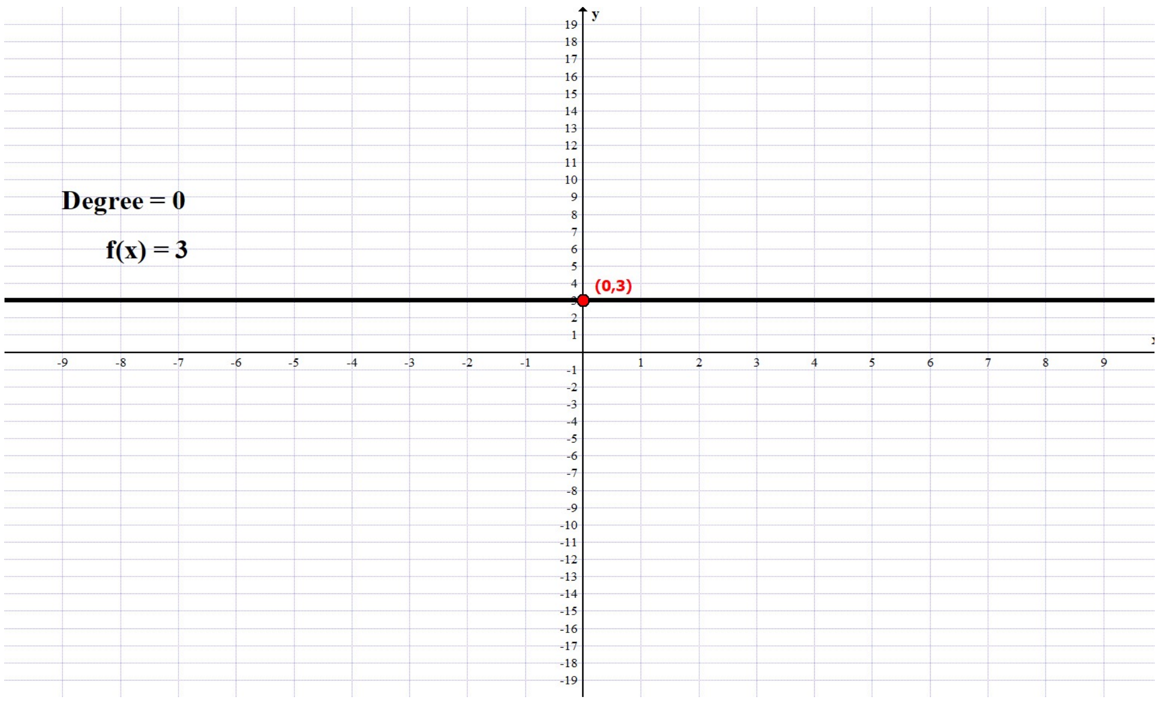
1. **n** is called the **degree** of the polynomial ; **deg f = n**
2. $\mathbf{a_0}$ is called the **constant term**
3. $\mathbf{a_n}$ is called the **leading coefficient**

Lower degree: $\mathbf{n} = 0, 1, 2$ We can find all their properties and graphs using techniques & skills we already possess.

Degree = 0: $\mathbf{n} = 0$

$\mathbf{y} = \mathbf{f}(\mathbf{x}) = \mathbf{a}_0 = \text{constant} = 3$ (for example) CONSTANT

Graph: One point & horizontal line



Degree = 1: $\mathbf{n} = 1$

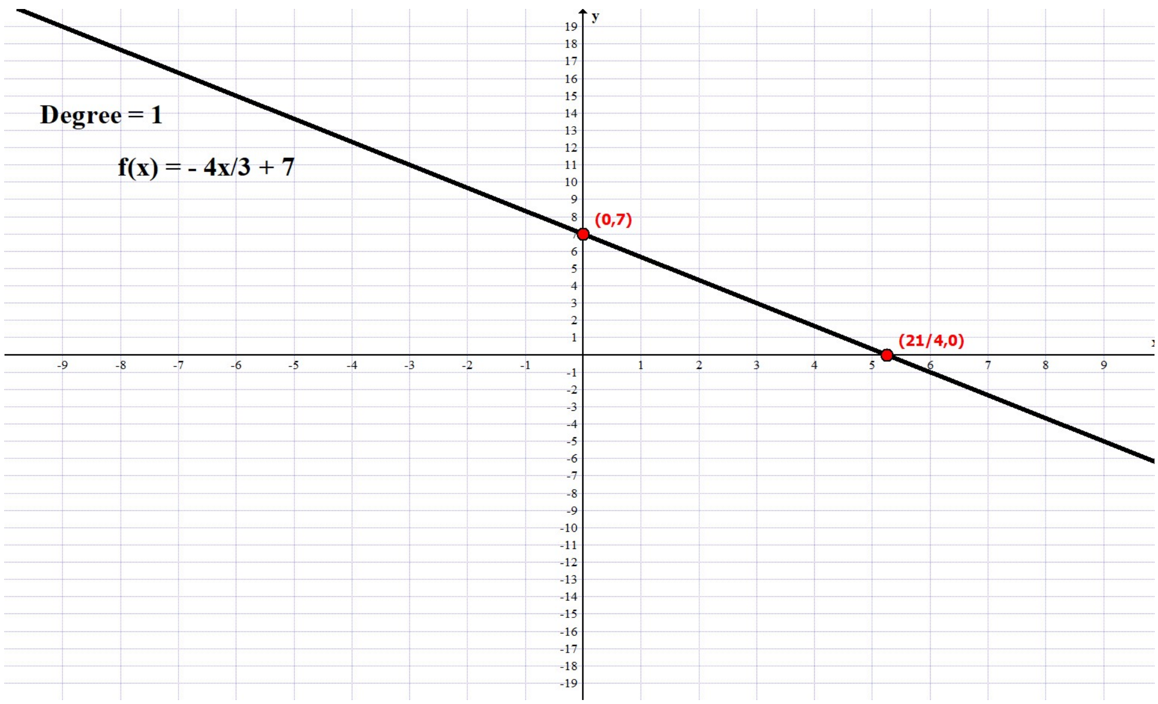
$\mathbf{y} = \mathbf{f}(\mathbf{x}) = \mathbf{a}_1\mathbf{x} + \mathbf{a}_0$ (usually written $\mathbf{y} = \mathbf{mx} + \mathbf{b}$) LINEAR

Consider $\mathbf{y} = \mathbf{f}(\mathbf{x}) = -\dfrac{4\mathbf{x}}{3} + 7$

$\mathbf{f}(0) = 7 \Rightarrow (0,7)$

$\mathbf{f}(\mathbf{x}) = 0 \Rightarrow \dfrac{4\mathbf{x}}{3} = 7 \Rightarrow \mathbf{x} = \dfrac{21}{4} \Rightarrow \left(\dfrac{21}{4}, 0\right)$

Graph: Two points & straight line



Degree = 2: $\mathbf{n}=2$ **; Parabola ; Quadratic**

$\mathbf{y}=\mathbf{f}(\mathbf{x})=\mathbf{a}_2\mathbf{x}^2+\mathbf{a}_1\mathbf{x}+\mathbf{a}_0$ (usually written $\mathbf{y}=\mathbf{a}\mathbf{x}^2+\mathbf{b}\mathbf{x}+\mathbf{c}$) Quadratic

Consider $\mathbf{y}=\mathbf{f}(\mathbf{x})=-3\mathbf{x}^2-12\mathbf{x}+4=4-12\mathbf{x}-3\mathbf{x}^2$

Graph: Five points (if nice) OR $\mathbf{h}(\mathbf{x})=\mathbf{A}(\mathbf{x}+\mathbf{C})^2+\mathbf{D}$; Complete the square

Example: 5 POINTS Method

$\mathbf{y}=\mathbf{f}(\mathbf{x})=-3\mathbf{x}^2-12\mathbf{x}+4=4-12\mathbf{x}-3\mathbf{x}^2$

$\mathbf{a}=-3$; $\mathbf{b}=-12$; $\mathbf{c}=4$

1. **Dom f** $=\mathbb{R}_{\mathbf{x}}=(-\infty,+\infty)_{\mathbf{x}}$
2. **Intercepts:**
   a. **y-intercept – Set x = 0 {Evaluate}:** $(0,\mathbf{c})=(0,4)$
   b. **x-intercept – Set y = f(x) = 0 {Solve}: Can QF {Quadratic Formula}**

$$\left(\frac{-\mathbf{b}\pm\sqrt{\mathbf{b}^2-4\mathbf{ac}}}{2\mathbf{a}},0\right)$$

$$\mathbf{x}=\frac{-\mathbf{b}\pm\sqrt{\mathbf{b}^2-4\mathbf{ac}}}{2\mathbf{a}}=\frac{-[-12]\pm\sqrt{[-12]^2-4[-3][4]}}{2[-3]}=\frac{12\pm\sqrt{192}}{-6}$$

$$=\frac{12\pm8\sqrt{3}}{-6}=-2\pm\frac{4}{3}\sqrt{3}\Rightarrow\mathbf{x}=-2-\frac{4}{3}\sqrt{3}\approx-4.31;\ \mathbf{x}=-2+\frac{4}{3}\sqrt{3}\approx0.309$$

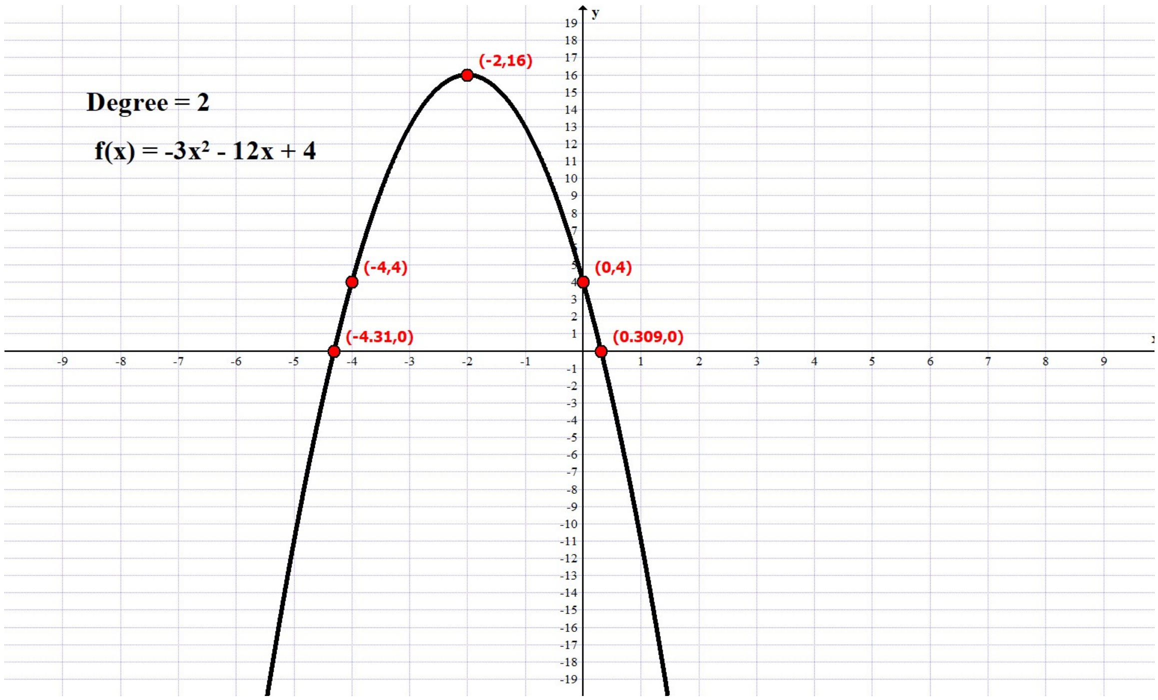
$\Rightarrow(-4.31,0)$; $(0.309,0)$

3. **Vertex:** $\left(-\frac{\mathbf{b}}{2\mathbf{a}},\mathbf{f}\left(-\frac{\mathbf{b}}{2\mathbf{a}}\right)\right)=\left(-\frac{\mathbf{b}}{2\mathbf{a}},\frac{4\mathbf{ac}-\mathbf{b}^2}{4\mathbf{a}}\right)$

$$\left(-\frac{\mathbf{b}}{2\mathbf{a}},\mathbf{f}\left(-\frac{\mathbf{b}}{2\mathbf{a}}\right)\right)=(-2,\mathbf{f}(-2))=(-2,16)\ ;\ \mathbf{f}(-2)=-3(-2)^2-12(-2)+4=16$$

4. **Symmetry Point: "Cheap" Point -** $\left(-\frac{b}{a}, c\right) = (-4, 4))$

5. **Range: Projection of graph onto the y-axis -** $(-\infty, 16\,]_y$



Example: $\mathbf{h(x) = A(x+C)^2 + D}$ **Method**

$\mathbf{f}(\mathbf{x}) = -3\mathbf{x}^2 - 12\mathbf{x} + 4 \Rightarrow$

$-3\mathbf{x}^2 - 12\mathbf{x} + 4 = -3(\mathbf{x}^2 + 4\mathbf{x} + \_\_\_\_\_) + 4 - \_\_\_\_\_\_$

$-3(\mathbf{x}^2 + 4\mathbf{x} + 4) + 4 + 12$ [Complete the square: 4 ; 2 ,4]

$\Rightarrow \mathbf{h}(\mathbf{x}) = -3(\mathbf{x}+2)^2 + 16 \Rightarrow$

$\mathbf{A} = -3$: Vertical stretch & reflection in x-axis

$\mathbf{B} = 1$: No effect

$\mathbf{C} = 2$: Horizontal translation 2 units to the left

$\mathbf{D} = 16$: Vertical translation 16 units upward

1. **Dom f** $= \mathbb{R}_{\mathbf{x}} = (-\infty, +\infty)_{\mathbf{x}}$
2. **Intercepts:**
   a. **y-intercept – Set x = 0 {Evaluate}:**
      $\mathbf{h}(0) = -3(2)^2 + 16 = 4 \Rightarrow (0,4)$
   b. **x-intercept – Set y = h(x) = 0 {Solve}:**

      $\mathbf{h}(\mathbf{x}) = -3(\mathbf{x}+2)^2 + 16 = 0 \Rightarrow 3(\mathbf{x}+2)^2 = 16$

      $\Rightarrow (\mathbf{x}+2)^2 = \frac{16}{3} \Rightarrow \mathbf{x} + 2 = \pm\sqrt{\frac{16}{3}} = \pm\frac{4}{\sqrt{3}} * \frac{\sqrt{3}}{\sqrt{3}} = \pm\frac{4}{3}\sqrt{3}$

      $\Rightarrow \mathbf{x} = -2 \pm \frac{4}{3}\sqrt{3} \Rightarrow \mathbf{x} = -2 - \frac{4}{3}\sqrt{3} \approx -4.31;\ \mathbf{x} = -2 + \frac{4}{3}\sqrt{3} \approx 0.309$

      $\Rightarrow (-4.31,0)\ ;\ (0.309,0)$

3. **Vertex:** $(-2,16)$
4. **Symmetry Point: "Cheap" Point -** $(-4,4))$
5. **Range: Projection of graph onto the y-axis -** $(-\infty,16\,]_{\mathbf{y}}$

See previous graph.

Stay tuned: Higher degree: $\mathbf{n} \geq 3$ require new techniques

