Polynomial FUNctions
Higher Degree

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Polynomials:

$\mathbf{y} = \mathbf{f}(\mathbf{x}) = \mathbf{a_n x^n} + \mathbf{a_{n-1} x^{n-1}} + \dots + \mathbf{a_3 x^3} + \mathbf{a_2 x^2} + \mathbf{a_1 x} + \mathbf{a_0}$; **n** where is non-negative integer & $\mathbf{a_n}, \mathbf{a_{n-1}}, \dots, \mathbf{a_2}, \mathbf{a_1}, \mathbf{a_0} \in \mathbb{R}$ (given)

Note:

1. **n** is called the **degree** of the polynomial ; **deg f = n**
2. $\mathbf{a}_0$ is called the **constant term**
3. $\mathbf{a_n}$ is called the **leading coefficient**

Lower degree: $\mathbf{n} = 0, 1, 2$ We have discussed these previously.

Higher degree: $\mathbf{n} \geq 3$

Note: Calculus is required to analyze the more complicated polynomials, but we will develop some techniques to study *some* polynomials of higher degree. In particular, we will *only* consider the following:

Note:

1. **Domain:** **Dom** $\mathbf{f}(\mathbf{x}) = \mathbb{R}_\mathbf{x}$ (ANY Polynomial)
2. **Intercept Points:**
   a. **y-intercept point:** $(0, \mathbf{f}(0)) = (0, \mathbf{a}_0)$ **; Evaluate**
   b. **x-intercept point(s):** $\mathbf{f}(\mathbf{x}) \overset{\text{SET}}{=} 0 \Rightarrow (?, 0)$ **; Solve**
3. **Continuity:** **Cont** $\mathbf{f}(\mathbf{x}) = \mathbb{R}_\mathbf{x}$ (ANY Polynomial) **- NO Breaks!**
4. **Behavior at Infinity ("End Behavior"): Analyze** $\mathbf{a_n x^n}$
   a. $\mathbf{x} \to +\infty \Rightarrow \mathbf{f}(\mathbf{x}) \to ?$
   b. $-\infty \leftarrow \mathbf{x} \Rightarrow \mathbf{f}(\mathbf{x}) \to ?$
5. **Range**
   Deg f $= 1, 3, 5, \dots$: **Range** $\mathbf{f}(\mathbf{x}) = \mathbb{R}_\mathbf{y}$
   Deg f $= 2, 4, 6, \dots$: **Range** $\mathbf{f}(\mathbf{x})$ is more complicated