Annuity (Present Value)
Amortization

MATH by Wilson
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Question: How much does one need to deposit in an ordinary annuity to receive equal fixed payments of a pre-determined amount for a fixed number of periods , with an ending balance of \$0.00. The initial deposit is called the Present Value (PV) and is given by

$$\mathbf{PV} = \mathbf{PMT}\left[\frac{1-(1+\mathbf{i})^{-\mathbf{n}}}{\mathbf{i}}\right]$$

where

PV = Present Value = Initial Deposit
PMT = equal pre-determined periodic payment at the end of each fixed period
i = interest rate per period in decimal form (usually not **r**, the annual rate)
n = number of equal payments (number of periods)

Example 01: David B wants to receive \$2500 per month for 15 years and can obtain a guaranteed annual interest rate of 3.6%. How much does he need to fund this annuity?

Solution:

We have

$$\mathbf{PV} = \mathbf{PMT}\left[\frac{1-(1+\mathbf{i})^{-\mathbf{n}}}{\mathbf{i}}\right] = \mathbf{PMT}\left[\frac{1-\left(1+\frac{\mathbf{r}}{100\,\mathbf{m}}\right)^{-\mathbf{m}*\mathbf{t}}}{\frac{\mathbf{r}}{100\,\mathbf{m}}}\right]$$

$$= \$2500\left[\frac{1-\left(1+\frac{3.6}{100*12}\right)^{-12*15}}{\frac{3.6}{100*12}}\right] = \$347{,}317$$

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Given a debt (house, automobile, boat, …) we want to pay off with equal payments at the end of a certain number of fixed periods, what payment will be required and how much interest will be paid?

The debt amount is just the PV in the formula above so we just use this formula solved for PMT:

$$\mathbf{PMT} = \mathbf{PV}\left[\frac{\mathbf{i}}{1-(1+\mathbf{i})^{-\mathbf{n}}}\right]$$

where

PV = Present Value = Initial Debt
PMT = equal periodic payment due at the end of each fixed period
i = interest rate per period in decimal form (usually not **r**, the annual rate)
n = number of equal payments (number of periods)

Example 02: Andrew J buys an auto for $3500 and finances it for 3 years with an annual rate of interest of 6 %.What will his payment be and how much interest will he pay for the privilege of qualifying for this loan?

Solution:

We have

$$\mathbf{PMT} = \mathbf{PV}\left[\frac{\mathbf{i}}{1-(1+\mathbf{i})^{-\mathbf{n}}}\right] = \mathbf{PV}\left[\frac{\frac{\mathbf{r}}{100\,\mathbf{m}}}{1-\left(1+\frac{\mathbf{r}}{100\,\mathbf{m}}\right)^{-\mathbf{m}*\mathbf{t}}}\right]$$

$$= \$3500\left[\frac{\frac{6}{100*12}}{1-\left(1+\frac{6}{100*12}\right)^{-12*3}}\right] = \$106.50$$

The interest paid is

$$\mathbf{Interest} = \mathbf{PMT}*\mathbf{m}*\mathbf{t} - \mathbf{PV} = \$333.20$$

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Example 03: Construct an amortization schedule for a debt of $1500 to be paid off in 6 months with an annual rate of 8 %.

Solution:

We have

$$\mathbf{PMT} = \mathbf{PV}\left[\frac{\mathbf{i}}{1-(1+\mathbf{i})^{-\mathbf{n}}}\right] = \$1500\left[\frac{\frac{8/12}{100}}{1-\left(1+\frac{8/12}{100}\right)^{-6}}\right] = \$255.90$$

The corresponding amortization schedule is given below:

		Finite Mathematics					
		PAYMENT (P&I)					
		$255.90					
		APR					
		8.00%					
				TOTAL			
				INTEREST			
				$35.19		BALANCE	
			PRINCIPLE			To	
Month		PAYMENT	REDUCTION	INTEREST		PAY	
0						$1,500.00	
1		$255.90	$245.90	$10.00		$1,254.10	
2		$255.90	$247.54	$8.36		$1,006.56	
3		$255.90	$249.19	$6.71		$757.37	
4		$255.90	$250.85	$5.05		$506.52	
5		$255.90	$252.52	$3.38		$254.00	
6		$255.69	$254.00	$1.69		$0.00	
SUM:		$1,535.19	$1,500.00				

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Month 1:

$$\text{Interest paid} = \text{Old Balance} * \frac{\mathbf{i}}{100} = \$1500 * \frac{8/12}{100} = 10.00$$

$$\text{New Balance} = \text{Old Balance} - (\text{Payment} - \text{Interest paid})$$

$$= \$15000 - (255.90 - 10.00) = \$1254.10$$

Month 2:

$$\text{Interest paid} = \text{Old Balance} * \frac{\mathbf{i}}{100} = \$1254.10 * \frac{8/12}{100} = \$8.36$$

$$\text{New Balance} = \text{Old Balance} - (\text{Payment} - \text{Interest paid})$$

$$= \$1254.10 - (\$255.90 - \$8.36) = \$1006.56$$

Month 3

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