Compound Interest
Interest does make Interest
More Algebra Required
(Exponential/Logarithmic properties)

MATH by Wilson
Your Personal Mathematics Trainer
MathByWilson.com

To get started, assume that we invest $1000 for 1 year at 6% simple interest. Then the interest we earn is

$$\mathbf{I} = \mathbf{Prt} = \$1000\left(\frac{6}{100\,\text{year}}\right)(1\,\text{year}) = \$60$$

Simple interest is paid one time. But what if the interest was paid twice per year? We would be paid ½ the yearly rate per period but the interest we make in the first period will be making interest during the second period:

Period	Principle	Interest for period	Total Interest
1	$1000	(1000)(3/100) = $30.00	$30.00
2	$1030	(1030)(3/100) = $30.90	$60.90

More Interest ($$$$$) for us! Yeah! What if we were paid 3 times per year (2% per period) Then

Period	Principle	Interest for period	Total Interest
1	$1000	(1000)(2/100) = $20.00	$20.00
2	$1020	(1020)(2/100) = $20.40	$40.40
3	$1040.40	(1040.40)(2/100) = $20.81	$61.21

Even more Interest ($$$$$) for us! BIGGER Yeah! This is Compound Interest – Interest making Interest! I LOVE IT!

Now assume that we invest an amount P at r % per year. If the number of compounding periods is m per year, then P will grow to an amount A (Future Value) given by the following formula:

 © 2026 MathByWilson.com

$$\mathbf{A} = \mathbf{P}\left(1 + \frac{\mathbf{r}}{100\,\mathbf{m}}\right)^{\mathbf{m}\,\mathbf{t}} \quad \left[\$\left(1 + \frac{\mathbf{r}\ /\ \text{year}}{100\,\mathbf{m}\ /\ \text{year}}\right)\right]^{(\mathbf{m}/\text{year})(\mathbf{t}\ \text{years})}$$

where t is the number of years we let the investment grow.

What we have is an equation with 5 variables and thus we must be given 4 of them in order to solve for the one not given (unknown). Here is where your algebra skills, including exponential and logarithmic skills, are critical!

Example 01: Sharon S invests $5000 at 5 %. What will this $5000 be worth in 10 years if the compounding is

a. Annually (1 period per year)1
b. Semi-annually (2 periods per year)
c. Quarterly (4 periods per year)
d. Monthly (12 periods per year)
e. Daily (365 periods per year)

Solution:

Annually: $\mathbf{A} = \mathbf{P}\left(1 + \frac{\mathbf{r}}{100\,\mathbf{m}}\right)^{\mathbf{m}\,\mathbf{t}} = 5000\left(1 + \frac{5}{100*1}\right)^{1*10} = \$8144$

Semi-annually: $\mathbf{A} = \mathbf{P}\left(1 + \frac{\mathbf{r}}{100\,\mathbf{m}}\right)^{\mathbf{m}\,\mathbf{t}} = 5000\left(1 + \frac{5}{100*2}\right)^{2*10} = \$8193$

Quarterly: $\mathbf{A} = \mathbf{P}\left(1 + \frac{\mathbf{r}}{100\,\mathbf{m}}\right)^{\mathbf{m}\,\mathbf{t}} = 5000\left(1 + \frac{5}{100*4}\right)^{4*10} = \$8218$

Monthly: $\mathbf{A} = \mathbf{P}\left(1 + \frac{\mathbf{r}}{100\,\mathbf{m}}\right)^{\mathbf{m}\,\mathbf{t}} = 5000\left(1 + \frac{5}{100*12}\right)^{12*10} = \$8235$

Daily: $\mathbf{A} = \mathbf{P}\left(1 + \frac{\mathbf{r}}{100\,\mathbf{m}}\right)^{\mathbf{m}\,\mathbf{t}} = 5000\left(1 + \frac{5}{100*365}\right)^{365*10} = \$8243$

If the Present Value P is invested at r % *compounded continuously* ("moment-by-moment"), then our Future Value formula, using higher Mathematics, becomes

 © 2026 MathByWilson.com

$$\mathbf{A = P e^{rt}} \quad \left[\$\mathbf{e}^{\left(\frac{\mathbf{r}}{100\,\text{year}}\right)(\mathbf{t}\,\text{years})} \right]$$

where $\mathbf{e} \approx 2.718281828459045...$ is the important irrational number presented in College Algebra.

The good news is that there are only 4 variables in this equation. The bad news is that we must frequently work with the exponential and logarithmic properties from College Algebra.

Example 02: If we invest \$2500 at 3.5 % (annual rate) , what will the Future Value be in 6 years?

Solution:

We have

$$\mathbf{A = P e^{rt}} = \$2500\,\mathbf{e}^{\left(\frac{3.5}{100\,\text{year}}\right)(6\,\text{years})} = \$3084.20$$

Example 03: How many years will it take for \$7500 to become \$10000 if the rate of return is 3.75 % compounded continuously?

Solution:

We have

$$\mathbf{A = P e^{rt}} \Rightarrow \mathbf{e^{rt}} = \frac{\text{A}}{\text{P}} \Rightarrow$$

$$\mathbf{r\,t} = \ln\left(\mathbf{e^{rt}}\right) = \ln\left(\frac{\text{A}}{\text{P}}\right) \Rightarrow$$

$$\mathbf{t} = \frac{\ln\left(\frac{\text{A}}{\text{P}}\right)}{\mathbf{r}} = \frac{\ln\left(\frac{\$10000}{\$7500}\right)}{\frac{3.75}{100\,\text{year}}} = 7.672\,\text{years}$$

 © 2026 MathByWilson.com

Example 04: How many years will it take for \$5000 to double (Future Value = \$10000) if the rate of return is 6 % compounded continuously?

Solution:

We have

$$\mathbf{A} = \mathbf{P}\,\mathbf{e}^{\mathbf{r}\,\mathbf{t}} \Rightarrow \mathbf{e}^{\mathbf{r}\,\mathbf{t}} = \frac{\mathrm{A}}{\mathrm{P}} \Rightarrow$$

$$\mathbf{r}\,\mathbf{t} = \ln\left(\mathbf{e}^{\mathbf{r}\,\mathbf{t}}\right) = \ln\left(\frac{\mathrm{A}}{\mathrm{P}}\right) \Rightarrow$$

$$\mathbf{t} = \frac{\ln\left(\frac{\mathrm{A}}{\mathrm{P}}\right)}{\mathbf{r}} = \frac{\ln\left(\frac{\$10000}{\$5000}\right)}{\frac{6}{100\,\mathrm{year}}} = \frac{\ln(2)}{\frac{6}{100\,\mathrm{year}}} = 11.5525\,\mathrm{years}$$

Note: In this example, the Present Value P is \$5000. However, it does not matter what P is. For an interest rate of 6 %, it will ALWAYS take 11.5525 years to double.

Assume we are quoted an annual interest rate of r %. If we are being compounded m times per year our interest is making interest and hence our actual rate of return is higher. It is called the Annual Percentage Rate (APR) and is given by

$$\mathbf{APR} = \left(1 + \frac{\mathbf{r}}{100\,\mathbf{m}}\right)^{\mathbf{m}} - 1$$

If we are being compounded continuously, the APR is even higher:

$$\mathbf{APR} = \mathbf{e}^{\mathbf{r}} - 1$$

 © 2026 MathByWilson.com

Example 05: What is the APR if the annual rate that is compounded monthly is 2.75 %?

Solution:

We have

$$\mathbf{APR} = \left(1 + \frac{\mathbf{r}}{100\,\mathbf{m}}\right)^{\mathbf{m}} - 1 = \left(1 + \frac{2.75}{100 * 12}\right)^{12} - 1 = 0.0278493 = 2.785\%$$

Example 06: If the APR is 5.25 %, what is the annual rate r if the interest is compounded quarterly?

Solution:

We have

$$\mathbf{APR} = \left(1 + \frac{\mathbf{r}}{100\,\mathbf{m}}\right)^{\mathbf{m}} - 1 \Rightarrow$$

$$\left(1 + \frac{\mathbf{r}}{100\,\mathbf{m}}\right)^{\mathbf{m}} = \mathbf{APR} + 1 \Rightarrow$$

$$1 + \frac{\mathbf{r}}{100\,\mathbf{m}} = \sqrt[\mathbf{m}]{\mathbf{APR} + 1} \Rightarrow$$

$$\frac{\mathbf{r}}{100\,\mathbf{m}} = \sqrt[\mathbf{m}]{\mathbf{APR} + 1} - 1 \Rightarrow$$

$$\mathbf{r} = 100 * \mathbf{m} * \left(\sqrt[\mathbf{m}]{\mathbf{APR} + 1} - 1\right) = 100 * 4 * \left(\sqrt[4]{\frac{5.25}{100} + 1} - 1\right) = 5.149\%$$

Example 07: What is the APR if the annual rate that is compounded continuously is 2.75 %?

Solution:

We have

$$\mathbf{APR} = \mathbf{e}^{\frac{\mathbf{r}}{100}} - 1 = \mathbf{e}^{\frac{2.75}{100}} - 1 = 2.788\%;\text{slightly higher than in Example 05.}$$

 © 2026 MathByWilson.com